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TUNING ONTOLOGY INTEROPERABILITY

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Abstract. The main contribution of this paper is the notion of *ontology space*, which allows us to move from an ontology-centric vision to a constellation-centric vision of the Web, where multiple ontologies and their interactions can be explicitly modeled and studied. This, in turn, allows us to study how OWL ontologies can interoperate, and, in particular, to provide two main results. The first is a formalization of the intended semantics of the OWL importing operator as *opaque semantics*. This result makes explicit the fact that the semantics of the importing ontology and of the imported ontology are independent. The second is the introduction of the notion of *transparent semantics*, which allows us to constrain the semantics of an ontology to be the same, on selected language elements, to that of a second reference ontology.

1 Introduction

Ontologies are conceptualizations of some domain which encode a view which is *common* to a set of different parties. What "common" exactly means is somehow immaterial and depends mainly on the specific application being considered. However it is a fact that the original philosophical request of a unique ontological description of the world has been given up, that the attempts to build a representation which is "common enough" have often provided a lot of leverage, at least from an engineering point of view, and that many success stories can be found in the literature, see for instance [Gen,DFvH02].[4] The feature which is key to the success of ontologies is that the common understanding they encode is the basis for the *sharing* of knowledge among different parties. Notice

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[4] It has been argued that the goal of building a single representation of the world is impossible in principle and not only very hard/ complex/ costly. This issue is not discussed here because out of the goals of this paper. The interested reader can find a discussion about these issues in [Giu93,GS94,GG01].

that, in general, these parties have different languages, different knowledge, different perspectives of the same problem domain, and also different goals which, in turn, super-impose still further constraints on their way to describe the world [BGvH+03].

Sharing and re-use are “old” concepts for the Computer Science community; think for instance about the notions of (software) library, of modularisation, and also of object oriented programming. In this work, as part of the software development process, *at design time* one can choose and re-use any of existing libraries/ modules/ objects. [MJaM03,Jar05] report very interesting work which follows very much this line of thought and which aims at ontology modularization and re-use. Following this tendency, OWL (Ontology Web Language) already provides mechanisms for a bottom up incremental construction of common views [MvH03]. In particular it provides a name-space naming mechanism, derived from XML, which allows us, when defining an ontology, to reuse objects (e.g., classes, roles, individuals) defined elsewhere; and also an importing operator `imports` which allows us to include an entire ontology as part of the current specification, very much in the same fashion as, in the standard software engineering practice, software modules are imported into other modules.

However the Semantic Web allows for more complex forms of sharing and re-use. In the Web, both the importing and the imported ontology co-exist and any new party can (re-)use an ontology which is being used by its own designer, and which may have been already imported by other ontologies, maybe for very different purposes.[5] Each such ontology has its intended semantics, which may somehow depend on the syntax and semantics of the imported ontologies, but where there is no a priori reason why importing an ontology means importing also its intended semantics. Thus, for instance, the concept Car in the ontology of `Ferrari` will be more specific that the same concept in a general ontology about cars. Still `Ferrari` may want to import this latter ontology implicitly changing the meaning of the concept Car (see however [BGvH+03] for the problems that may arise in situations like this).

The main contribution of this paper is the notion of *ontology space*, also called *OWL space*, which allows us to formalize the presence of multiple ontologies. With OWL spaces we move from an ontology-centric vision (as it is in the current semantics of OWL, which considers a *single* ontology at the time.) to a constellation-centric vision where all the ontologies are considered and their interactions are explicitly modeled. This, in turn, allows us to provide two main results. The first is a formalization of the intended semantics of the OWL importing operator as *opaque semantics*. This result makes explicit the fact that the semantics of the importing ontology and of the imported ontology are independent. The second is the introduction of the notion of *transparent semantics*, which allows us to constrain the semantics of one ontology to be the same, on selected language elements, to that of a second reference ontology.

[5] This shift is similar in spirit to that from modules/objects to the situation where there are systems which are publicly available and which, *at run time*, answer to whoever sends them a request (as it happens, for instance, with Web Services).

The paper is structured as follows. Section 2 reviews the basic notion of OWL ontology. The main contribution here are the notions of *local* and of *foreign languages* which distinguish between local language elements defined within an ontology (i.e., within the same namespace) and all the others. Section 3 introduces the notion of *ontology space* and of *distributed intepretation*. Distributed interpretations allow for one (local) interpretation for each ontology in a OWL space and are at the basis of the main result of this section, which proves that, once performed the import closure, the semantics of an ontology are independent of the OWL space it is immersed in. This is the *opaque semantics* of the OWL importing operator. Finally, Section 4 introduces the notion of *transparent interpretation*. The related work and conclusions end the paper.

2 OWL Ontologies

In this section we recall the main concepts about OWL that are relevant for the rest of this document. For the sake of readability, we slightly simplify the presentation of such concepts, without loosing the main properties.

The OWL language is a W3C recommendation for expressing ontologies in the Semantic Web. OWL has three increasingly expressive sub-languages: OWL Lite, OWL DL and OWL Full. *OWL Lite* and *OWL DL*[6] are, like DAML+OIL, basically very expressive description logics; they are almost[7] equivalent to the $\mathcal{SHIF}(\mathbf{D}^+)$ and $\mathcal{SHOIN}(\mathbf{D}^+)$ DLs. OWL Full is clearly undecidable because it does not impose restrictions on the use of transitive properties. In this paper, we concentrate on the OWL DL (and therefore its sub-language OWL Lite) fragment of OWL. Detailed discussions on the relationship between OWL DL and OWL Full can be found in [Pan04].

According to [SWM04], an OWL ontology (hereafter ontology) begins with a namespace declaration. An *XML namespace* (or simply *namespace*), identified by a URI reference (URIref) [Gro01], is a collection of names [BHL99]. In a namespace declaration, namespace URIrefs (e.g. `http://www.car.org/car#`) are associated with namespace prefixes (e.g. `car`), which can be used as the shortened forms of namespace URIrefs. Accordingly resources (such as `http://www.car.org/car#Driver`), can have shortened forms (such as car:Driver). In this paper, we call namespace URIrefs and their corresponding prefixes *namespace identifiers*. A typical namespace declaration is similar to the following.

```
Namespace (xmlns:base = <http://www.car.org/sport_car#>
          xmlns:s_car = <http://www.car.org/sport_car#>
            xmlns:car = <http://www.car.org/car#>
            xmlns:abc = <http://www.abc.uk/market#>
            xmlns:xyz = <http://www.xyz.it/vehicle#>)
```

The first declaration identifies the namespace (through the base URIref) associated with the current ontology. The second one identifies the namespace URIref

[6] 'DL' for Description Logic.

[7] They also provide annotation properties, which Description Logics don't.

associated with the current ontology with the prefix `s_car`. The third, forth and fifth declarations identify the namespace URIrefs associated with supporting ontologies.

An OWL ontology consists of a set of *axioms*, including concept axioms, role axioms and individual axioms.[8] Let $\mathbf{C}$, $\mathbf{R}$ and $\mathbf{I}$ be the sets of URIrefs that can be used to denote concepts, roles and individuals respectively. The disjoint union of $\mathbf{C}$, $\mathbf{R}$ and $\mathbf{I}$ is denoted with $\mathbf{L}$. Table 3 presents the abstract syntax, DL syntax and semantics of OWL axioms, where $\mathsf{A} \in \mathbf{C}$ is a concept URIref, $C_1, \ldots, C_n$ are concept descriptions, $r \in \mathbf{R}$ is a role URIref, $r_1, \ldots, r_n$ are role descriptions and $\mathsf{o}, \mathsf{o}_1, \ldots, \mathsf{o}_n \in \mathbf{I}$ are individual URIrefs. Table 1 and 2 present the abstract syntax, DL syntax and semantics of OWL concept descriptions and abstract role descriptions, where $\sharp$ denotes cardinality. To simplify the presentation, in this paper we only consider the abstract domain and leave the discussion of the datatype domain [PH05] as one of our future works.

Abstract Syntax	DL Syntax	Semantics
Class(A)	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
Class(owl:Thing)	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
Class(owl:Nothing)	$\bot$	$\bot^{\mathcal{I}} = \emptyset$
intersectionOf($C_1, C_2, \ldots$)	$C_1 \sqcap C_2$	$(C_1 \sqcap C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
unionOf($C_1, C_2, \ldots$)	$C_1 \sqcup C_2$	$(C_1 \sqcup C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
complementOf(C)	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
oneOf($\mathsf{o}_1, \mathsf{o}_2, \ldots$)	$\mathsf{o}_1 \sqcup \mathsf{o}_2$	$(\mathsf{o}_1 \sqcup \mathsf{o}_2)^{\mathcal{I}} = \{\mathsf{o}_1{}^{\mathcal{I}}, \mathsf{o}_2{}^{\mathcal{I}}\}$
restriction(r someValuesFrom(C))	$\exists r.C$	$(\exists r.C)^{\mathcal{I}} = \{x \mid \exists y.\langle x, y\rangle \in r^{\mathcal{I}} \land y \in C^{\mathcal{I}}\}$
restriction(r allValuesFrom(C))	$\forall r.C$	$(\forall r.C)^{\mathcal{I}} = \{x \mid \forall y.\langle x, y\rangle \in r^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$
restriction(r hasValue(o))	$\exists r.\{\mathsf{o}\}$	$(\exists r.\{\mathsf{o}\})^{\mathcal{I}} = \{x \mid \langle x, \mathsf{o}^{\mathcal{I}}\rangle \in r^{\mathcal{I}}\}$
restriction(r minCardinality(m))	$\geqslant m\ r$	$(\geqslant m\ r)^{\mathcal{I}} = \{x \mid \sharp\{y.\langle x, y\rangle \in r^{\mathcal{I}}\} \geq m\}$
restriction(r maxCardinality(m))	$\leqslant m\ r$	$(\leqslant m\ r)^{\mathcal{I}} = \{x \mid \sharp\{y.\langle x, y\rangle \in r^{\mathcal{I}}\} \leq m\}$

Table 1. OWL Class Descriptions

Abstract Syntax	DL Syntax	Semantics
ObjectProperty(r)	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
ObjectProperty(s inverseOf(r))	r^{-}	$(r^{-})^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Table 2. OWL Abstract Property Descriptions

An OWL ontology can import (other) OWL ontologies, with the help of imports annotations similar to the following.

[8] Individual axioms are also called *facts*.

Abstract Syntax	DL Syntax	Semantics
(Class A partial $C_1\ \ldots\ C_n$)	$\mathsf{A} \sqsubseteq C_1 \sqcap \cdots \sqcap C_n$	$\mathsf{A}^{\mathcal{I}} \subseteq C_1^{\mathcal{I}} \cap \cdots \cap C_n^{\mathcal{I}}$
(Class A complete $C_1\ \ldots\ C_n$)	$\mathsf{A} \equiv C_1 \sqcap \cdots \sqcap C_n$	$A^{\mathcal{I}} = C_1^{\mathcal{I}} \cap \cdots \cap C_n^{\mathcal{I}}$
(EnumeratedClass A $\mathsf{o}_1\ \ldots\ \mathsf{o}_n$)	$\mathsf{A} \equiv \{\mathsf{o}_1\} \sqcup \ldots \sqcup \{\mathsf{o}_n\}$	$\mathsf{A}^{\mathcal{I}} = \{\mathsf{o}_1^{\mathcal{I}}, \ldots, \mathsf{o}_n^{\mathcal{I}}\}$
(SubClassOf C_1, C_2)	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$
(EquivalentClasses $C_1\ \ldots\ C_n$)	$C_1 \equiv \cdots \equiv C_n$	$C_1^{\mathcal{I}} = \cdots = C_n^{\mathcal{I}}$
(DisjointClasses $C_1\ \ldots\ C_n$)	$C_i \sqsubseteq \neg C_j$,	$C_1^{\mathcal{I}} \cap C_n^{\mathcal{I}} = \emptyset$,
	$(1 \leq i < j \leq n)$	$(1 \leq i < j \leq n)$
(SubPropertyOf r_1, r_2)	$r_1 \sqsubseteq r_2$	$r_1^{\mathcal{I}} \subseteq r_2^{\mathcal{I}}$
(EquivalentProperties $r_1\ \ldots\ r_n$)	$r_1 \equiv \cdots \equiv r_n$	$r_1^{\mathcal{I}} = \cdots = r_n^{\mathcal{I}}$
ObjectProperty(r super(r_1) ... super(r_n)	$r \sqsubseteq r_i$	$r^{\mathcal{I}} \subseteq r_i^{\mathcal{I}}$
domain(C_1) ... domain(C_k)	$\geqslant 1r \sqsubseteq C_i$	$r^{\mathcal{I}} \subseteq C_i^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
range(C_1) ... range(C_h)	$\top \sqsubseteq \forall r.C_i$	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times C_i^{\mathcal{I}}$
[Symmetric]	$r \equiv r^-$	$r^{\mathcal{I}} = (r^-)^{\mathcal{I}}$
[Functional]	$\mathsf{Func}(r)$	$\{\langle x,y\rangle \mid \sharp\{y.\langle x,y\rangle \in r^{\mathcal{I}}\} \leq 1\}$
[InverseFunctional]	$\mathsf{Func}(r^-)$	$\{\langle x,y\rangle \mid \sharp\{y.\langle x,y\rangle \in (r^-)^{\mathcal{I}}\} \leq 1\}$
[Transitive])	Trans(r)	$r^{\mathcal{I}} = (r^{\mathcal{I}})^+$
Annotationproperty(r)		
Individual(o type(C_1) ... type(C_n)	$C_i(\mathsf{o}), 1 \leq i \leq n$	$\mathsf{o}^{\mathcal{I}} \in C_i^{\mathcal{I}}, 1 \leq i \leq n$
value(r_1, o_1) ... value(r_n, o_n)	$r_i(\mathsf{o}, \mathsf{o}_i)$,$1 \leq i \leq n$	$\langle \mathsf{o}^{\mathcal{I}}, \mathsf{o}_i^{\mathcal{I}}\rangle \in r^{\mathcal{I}}, 1 \leq i \leq n$
Sameindividual($\mathsf{o}_1\ \ldots\ \mathsf{o}_n$)	$\mathsf{o}_1 = \cdots = \mathsf{o}_n$	$\mathsf{o}_1^{\mathcal{I}} = \cdots = \mathsf{o}_n^{\mathcal{I}}$
DifferentIndividuals($\mathsf{o}_1\ \ldots\ \mathsf{o}_n$)	$\mathsf{o}_i \neq \mathsf{o}_j, 1 \leq i < j \leq n$	$\mathsf{o}_i^{\mathcal{I}} \neq \mathsf{o}_j^{\mathcal{I}}, 1 \leq i < j \leq n$

Table 3. OWL Axioms

```
Annotation(imports <http://www.car.org/car#>)
Annotation(imports <http://www.abc.uk/market#>)
Annotation(imports <http://www.xyz.it/vehicle#>)
```

These imports annotations include all the axioms in the three supporting ontologies into the current ontology. Notice the distinction of the namespace and imports mechanisms: the former one provides a convenient way to reference names (URIrefs) from other ontologies, while the latter one is provided to indicate the intention to include the axioms in the targeted ontologies. In order to make best use of imported ontologies, they would normally be coordinated with a namespace declaration [SWM04]. Note that most aspects of the Web, including missing, unavailable, and time-varying documents, reside outside the namespace and imports mechanisms [PSHH04].

Formally, an ontology is defined as follows.

Definition 1 (Ontology). *An* OWL Ontology *(or simply an* ontology*) is a pair $\mathcal{W}_{\mathtt{i}} = \langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}}\rangle$, where* $\mathtt{i}$ *is the namespace identifier of the ontology $\langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}}\rangle$, $\mathcal{M}_{\mathtt{i}}$ (called the* imports-box*) is the set of ontology namespace identifiers which identify the set of ontologies that are imported by $\mathcal{W}_{\mathtt{i}}$, and $\mathcal{O}_{\mathtt{i}}$ contains a set of class, property and individual axioms.* ⋄

For instance the ontology $\langle \mathcal{M}_{\texttt{s\_car}}, \mathcal{O}_{\texttt{s\_car}} \rangle$, where $\mathcal{M}_{\texttt{s\_car}} = \{\texttt{car}, \texttt{xyz}\}$, the following are examples of class descriptions that can appear in $\mathcal{O}_{\texttt{s\_car}}$:

$$\texttt{s\_car}{:}\mathsf{C},\ \texttt{abc}{:}\mathsf{E},\ (\texttt{s\_car}{:}\mathsf{C}) \sqcap (\texttt{car}{:}\mathsf{D}),\ \exists(\texttt{s\_car}{:}r).(\texttt{s\_car}{:}\mathsf{C}) \sqcup (\texttt{xyz}{:}\mathsf{F}). \quad (1)$$

Note that we can use URIrefs (such as $\texttt{abc}{:}\mathsf{E}$) which are neither locally defined nor imported.

Definition 2 (Local Language). *Given an OWL ontology $\mathcal{W}_{\texttt{i}} = \langle \mathcal{M}_{\texttt{i}}, \mathcal{O}_{\texttt{i}} \rangle$, a* local class *w.r.t. $\mathcal{W}_{\texttt{i}}$ is an element of $\mathbf{C}$ that is declared in $\mathcal{O}_{\texttt{i}}$ with the namespace identifier* $\texttt{i}$. Local properties *and* local individuals *are defined analogously. The set of local class, local properties and local individuals of $\mathcal{W}_{\texttt{i}}$ are denoted by $\mathbf{C}_i$, $\mathbf{R}_i$ and $\mathbf{I}_i$. The* local language *of $\mathcal{W}_{\texttt{i}}$, i.e., $\mathbf{L}_{\texttt{i}}$, is the disjoint union of them. Elements of a local language are called* local URIrefs*.* $\diamond$

Definition 3 (Foreign Language). *Any class URIref of the form* $\texttt{j}{:}\texttt{x}$ *that occurs in $\mathcal{O}_{\texttt{i}}$ which is not in $\boldsymbol{L}_{\texttt{i}}$, is called a $\texttt{j}$-foreign class of $\mathcal{W}_{\texttt{i}}$. $\texttt{j}$-foreign roles and $\texttt{j}$-foreign individuals are defined analogously, The union of the $\texttt{j}$-foreign classes, properties, and individuals, forms the $\texttt{j}$-foreign language. The union of the $\texttt{j}$-foreign languages, for every $\texttt{j}$, forms the* foreign language *of $\mathcal{W}_{\texttt{i}}$. We abuse the terminology here and call elements of a $\texttt{j}$-foreign language* $\texttt{j}$-foreign URIrefs*.* $\diamond$

Intuitively, the local language of $\langle \mathcal{M}_{\texttt{i}}, \mathcal{O}_{\texttt{i}} \rangle$ is the set of class, property, and individual URIrefs introduced by the local ontology $\mathcal{O}_{\texttt{i}}$. For instance, $\texttt{s\_car}{:}\mathsf{C}$ and $\texttt{s\_car}{:}r$ from (1) are in the local language of the ontology $\langle \mathcal{M}_{\texttt{s\_car}}, \mathcal{O}_{\texttt{s\_car}} \rangle$, viz. $\mathbf{L}_{\texttt{s\_car}}$.

Example 1. Assume that the local ontology $\mathcal{O}_{\texttt{car}}$ of $\mathcal{W}_{\texttt{car}}$ contains the axioms

$$\texttt{car}{:}\mathsf{Car} \sqsubseteq \texttt{xyz}{:}\mathsf{Vehicle} \quad \text{and} \quad \texttt{car}{:}\mathsf{FastCar} \sqsubseteq \texttt{car}{:}\mathsf{Car}$$

that the local ontology Ontology $\mathcal{O}_{\texttt{s\_car}}$ of $\mathcal{W}_{\texttt{s\_car}}$ contains the following axiom

$$\texttt{s\_car}{:}\mathsf{SportCar} \sqsubseteq \texttt{car}{:}\mathsf{FastCar} \sqcap \texttt{abc}{:}\mathsf{ExpensiveGood}$$

and that $\mathcal{W}_{\texttt{s\_car}}$ imports $\mathcal{W}_{\texttt{car}}$ and $\mathcal{W}_{\texttt{car}}$ imports $\mathcal{W}_{\texttt{xyz}}$. According to the above definition, we have:

1. $\texttt{abc}{:}\mathsf{ExpensiveGood}$ is a $\texttt{abc}$-foreign class w.r.t. $\mathcal{W}_{\texttt{s\_car}}$ because $\texttt{abc}{:}\mathsf{ExpensiveGood}$ appears in $\mathcal{O}_{\texttt{s\_car}}$;
2. $\texttt{car}{:}\mathsf{FastCar}$ is a $\texttt{car}$-foreign class w.r.t. $\mathcal{W}_{\texttt{s\_car}}$ because $\texttt{car} \in \mathcal{M}_{\texttt{s\_car}}$ and $\texttt{car}{:}\mathsf{FastCar} \in \mathbf{L}_{\texttt{car}}$;
3. $\texttt{xyz}{:}\mathsf{Vehicle}$ is a $\texttt{xyz}$-foreign class w.r.t. $\mathcal{W}_{\texttt{car}}$ because $\texttt{xyz} \in \mathcal{M}_{\texttt{car}}$ and $\texttt{xyz}{:}\mathsf{Vehicle} \in \mathbf{L}_{\texttt{xyz}}$; since $\texttt{car} \in \mathcal{M}_{\texttt{s\_car}}$, we have $\texttt{xyz}{:}\mathsf{Vehicle}$ is a $\texttt{xyz}$-foreign class w.r.t. $\mathcal{W}_{\texttt{s\_car}}$.

3 Ontology Spaces

There are two motivations of introducing ontology spaces. Firstly, although an ontology (from the epistemological point of view) is believed to be a unique model for a domain, we have to handle related ontologies introduced by URIref and the imports mechanisms. Secondly, we introduce some requirements to make best use of the imports mechanism. As described in the last section, OWL imports annotations do not guarantee the existence of imported ontologies. Similarly, the occurrence of an URIrefs in an axiom does not impose requirement of the existence of an ontological resource associated with such an URIref. For instance, the axiom $(\mathtt{car} : \mathsf{Car}) \sqsubseteq (\mathtt{abc} : \mathsf{ExpensiveGood})$ occurring in the ontology $\mathcal{W}_{\mathtt{car}}$, does not guarantee the existence of the ontology $\mathcal{W}_{\mathtt{abc}}$; furthermore, even $\mathcal{W}_{\mathtt{abc}}$ does exist, this axiom does not impose the requirement that $\mathsf{ExpensiveGood}$ is in the local language of $\mathcal{W}_{\mathtt{abc}}$, viz. $\mathbf{L}_{\mathtt{abc}}$. When we consider an ontology space, we impose these requirements.

Definition 4 (Ontology Space). *Let I be a set of ontology namespace identifiers. An* ontology space *on I is a family of ontologies $\mathfrak{S}_I = \{\mathcal{W}_{\mathtt{i}} = \langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}} \rangle\}_{i \in I}$ such that $\mathcal{M}_{\mathtt{i}} \subseteq I$, for each $\mathtt{i} \in I$, and each $\mathtt{j}$-foreign URIref $\mathtt{j:x}$ occurring in $\mathcal{O}_{\mathtt{i}}$, is contained in the local language of $\mathcal{W}_{\mathtt{j}}$.* $\diamond$

Note that, given an OWL space $\{\langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}} \rangle\}_{\mathtt{i} \in I}$, the above definition requires that: (i) all the imported ontologies of each ontology exist and are in $\{\langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}} \rangle\}_{\mathtt{i} \in I}$, and (ii) foreign URIrefs of each ontology in $\{\langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}} \rangle\}_{\mathtt{i} \in I}$ should be introduced by some ontologies in $\{\langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}} \rangle\}_{\mathtt{i} \in I}$. Based on an OWL space, we introduce the concept of imports closure.

Definition 5 (Imports Closure). *Let $\mathfrak{S}$ be an OWL space. The* import closure *of $\mathcal{W}_{\mathtt{i}}$ w.r.t. $\mathfrak{S}_I$ is written as $\mathcal{W}_{\mathtt{i}}^{\mathfrak{S}}$, recursively defined as follows:*

1. *If $\mathcal{O}_{\mathtt{i}} \subseteq \mathcal{W}_{\mathtt{i}}^{\mathfrak{S}}$;*
2. *if $\phi \in \mathcal{W}_{\mathtt{j}}^{\mathfrak{S}}$ and $j \in \mathcal{M}_{\mathtt{i}}$ then $\phi \in \mathcal{W}_{\mathtt{i}}^{\mathfrak{S}}$;*
3. *nothing else is in $\mathcal{W}_{\mathtt{i}}^{\mathfrak{S}}$.* $\diamond$

Example 2. Consider the ontology space $\mathfrak{S} = \{\mathcal{W}_1, \mathcal{W}_2\}$, where $\mathcal{W}_1 = \langle \mathcal{M}_1 = \{2\}, \mathcal{O}_1 = \{1{:}\mathsf{A} \sqsubseteq 2{:}\mathsf{B}\} \rangle$ and $\mathcal{W}_2 = \langle \mathcal{M}_2 = \{1\}, \mathcal{O}_2 = \{2{:}\mathsf{B} \sqsubseteq 1{:}\mathsf{C}\} \rangle$, which are depicted as follows:

$$\begin{array}{cc} \mathcal{M}_1 = \{2\} & \mathcal{M}_2 = \{1\} \\ \mathcal{O}_1 = \langle \{1{:}\mathsf{A} \sqsubseteq 2{:}\mathsf{B}\}, \emptyset, \emptyset \rangle & \mathcal{O}_2 = \langle \{2{:}\mathsf{B} \sqsubseteq 1{:}\mathsf{C}\}, \emptyset, \emptyset \rangle \end{array}$$

We have $\mathcal{W}_1^{\mathfrak{S}} = \mathcal{W}_2^{\mathfrak{S}} = \langle \{1{:}\mathsf{A} \sqsubseteq 2{:}\mathsf{B},\ 2{:}\mathsf{B} \sqsubseteq 1{:}\mathsf{C}\}, \emptyset, \emptyset \rangle$.

Definition 6 (Local and Distributed Interpretation). *Let $\mathfrak{S}_I$ be an ontology space on I. An interpretation $\mathcal{I}_{\mathtt{i}}$ is called* local interpretation *of $\mathcal{W}_{\mathtt{i}}$, written $\mathcal{I}_{\mathtt{i}} \models \mathcal{W}_{\mathtt{i}}$, iff $\mathcal{I}_{\mathtt{i}}$ satisfies all the axioms in $\mathcal{W}_{\mathtt{i}}^{\mathfrak{S}_I}$. A family of interpretations $\hat{\mathcal{I}} = \{\mathcal{I}_{\mathtt{i}}\}_{\mathtt{i} \in I}$ is called a* distributed interpretation *of $\mathfrak{S}_I$, written $\hat{\mathcal{I}} \models \mathfrak{S}_I$, iff, for each $\mathtt{i} \in I$, $\mathcal{I}_{\mathtt{i}} \models \mathcal{W}_{\mathtt{i}}$.* $\diamond$

Example 3. Consider a modified version of the ontology space of Example 2 (depicted as follows).

$$\mathcal{M}_1 = \{2\} \qquad\qquad \mathcal{M}_2 = \emptyset$$
$$\mathcal{O}_1 = \left\{\begin{array}{l}\mathsf{1{:}A} \sqsubseteq \mathsf{2{:}B},\\ \mathsf{1{:}C} \sqsubseteq \mathsf{1{:}D}\end{array}\right\} \qquad\qquad \mathcal{O}_2 = \{\mathsf{2{:}B} \sqsubseteq \mathsf{1{:}C}\}$$

An example of distributed interpretation $\hat{\mathcal{I}}_{12}$ of the above OWL space is composed of two interpretations $\mathcal{I}_1$ and $\mathcal{I}_2$ on two distinct domains $\Delta^{\mathcal{I}_1} = \{a, b, c, d\}$ and $\Delta^{\mathcal{I}_2} = \{x, y, z, w\}$.

$\mathtt{i}{:}\mathsf{X}$	$(\mathtt{i}{:}\mathsf{X})^{\mathcal{I}_1}$	$(\mathtt{i}{:}\mathsf{X})^{\mathcal{I}_2}$
1:A	$\{a\}$	Not defined
2:B	$\{a, b\}$	$\{x, y\}$
1:C	$\{a, b, c\}$	$\{x, y, z\}$
1:D	$\{a, b, c, d\}$	Not defined

Notice that, in the previous example the classes 2:B and 1:C, are interpreted by $\mathcal{I}_1$ and $\mathcal{I}_2$ in two different sets of individuals. For this reason this $\hat{\mathcal{I}}_{12}$ we say that it provides *opaque interpretation* of 2:B and 1:C.

Notice that this happens even if $\mathcal{W}_1$ imports $\mathcal{W}_2$. This highlights the fact that in importing an ontologies we are importing only the axioms and not the interpretation. We are still free enough to interpret the imported language, in our own local domain. The only constraint imposed by the import is that we have to satisfy all the imported axioms.

In general in the Semantic Web classes, opaque interpretation of properties and individuals are interpreted is an acceptable hypothesis. Indeed ontologies are usually independently developed by different users, and therefore with a different intended semantics. This coincides with the standard semantics given in the W3C, where semantics is given with respect to the single ontology. In the following theorem we show that opaque semantics in ontology spaces coincides with the W3C semantics, where imports statements is interpreted as axioms inclusion.

Definition 7 (Opaque Entailment). *Let $\mathfrak{S}$ be an ontology space on I, ϕ a concept axiom, a role axiom or an individual axiom. We say that $\mathfrak{S}$* entails *ϕ in the ontology $\mathcal{W}_{\mathtt{i}}$ ($\mathtt{i} \in I$), written $\mathfrak{S} \models_{\mathtt{i}} \phi$, if, for every distributed interpretation $\mathcal{I}_{\mathtt{i}}$ of $\mathfrak{S}$, $\mathcal{I}_{\mathtt{i}} \models \phi$.* ◇

Theorem 1. *$\mathfrak{S} \models_i \phi$ if and only if $\mathcal{W}_{\mathtt{i}}^{\mathfrak{S}} \models \phi$.*

The above theorem says that, once we have computed the imports closure, the ontology embedded in the ontology space, behaves exactly as the ontology in isolation.

4 Tuning the Transparency among Ontologies

In this section, we introduce *partially transparent ontologies*. Intuitively, in a partially transparent ontology, we can share the interpretation of some individual URIrefs (as well as some class and property URIrefs related to these individuals) with other ontologies.

Definition 8 (Partially Transparent Ontologies). *A partially transparent ontology (PTO) is a tuple $\mathcal{W}_{\mathtt{i}} = \langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}}, \mathcal{S}_{\mathtt{i}} \rangle$, where* $\mathtt{i}$ *is the namespace identifier of $\mathcal{W}_{\mathtt{i}}$, $\mathcal{M}_{\mathtt{i}}$ is an imports-box, $\mathcal{O}_{\mathtt{i}}$ is a set of class, property and individual axioms, and $\mathcal{S}_{\mathtt{i}}$ (called the* share-box*) is a set of tuples of the form $\langle \mathtt{s}, \mathbf{N}_{\mathtt{s}} \rangle$, where* $\mathtt{s}$ *is an ontology namespace identifier and $\mathbf{N}_{\mathtt{s}}$ is a set of* $\mathtt{s}$*-foreign URIrefs, called* shared URIrefs *from ontology $\mathcal{W}_{\mathtt{s}}$. We use the notation $[\mathcal{S}_{\mathtt{i}}]$ to represent the set of shared namespace identifiers in $\mathcal{S}_{\mathtt{i}}$.* $\diamond$

Note that if $\mathcal{W}_{\mathtt{i}}$ imports the shared ontologies $\mathcal{W}_{\mathtt{s}}$, i.e. $\mathtt{s} \in \mathcal{M}_{\mathtt{i}}$, then the inputs would overwrite sharing. In this case, $\mathcal{W}_{\mathtt{i}}$ will share *all* the individual URIrefs from $\mathcal{W}_{\mathtt{s}}$, instead of just *some* of them (which are specified in the share-box). For convenience, we sometimes ignore the share-box of a PTO if the share-box is empty; i.e., we can write $\langle \mathcal{M}_{\mathtt{j}}, \mathcal{O}_{\mathtt{j}}, \emptyset \rangle$ as $\langle \mathcal{M}_{\mathtt{j}}, \mathcal{O}_{\mathtt{j}} \rangle$.

Example 4. Here are two example TPOs $\mathcal{W}_{\mathtt{abcCar}}$ and $\mathcal{W}_{\mathtt{xyz}}$, where $\mathcal{W}_{\mathtt{abcCar}} = \langle \mathcal{M}_{\mathtt{abcCar}} = \emptyset, \mathcal{O}_{\mathtt{abcCar}} = \{\exists(\mathtt{abcCar}{:}\mathit{order}).\top \sqsubseteq \mathtt{abcCar}{:}\mathsf{Customer}, \mathtt{abcCar}{:}\mathsf{Customer}(\mathtt{abcCar{:}John}), \mathtt{abcCar}{:}\mathit{order}(\mathtt{abcCar{:}Alice}, \mathtt{abcCar{:}car1})\}\rangle$ and $\mathcal{W}_{\mathtt{xyz}} = \langle \mathcal{M}_{\mathtt{xyz}} = \emptyset, \mathcal{O}_{\mathtt{xyz}} = \{\mathtt{abcCar}{:}\mathsf{Customer}(\mathtt{xyz{:}David})\}, \mathcal{S}_{\mathtt{xyz}} = \{\langle \mathsf{abcCar}, \{\mathtt{abcCar{:}Alice}, \mathtt{abcCar}{:}\mathsf{Customer}\}\rangle\}\rangle$. Note that $\mathcal{W}_{\mathtt{xyz}}$ shares $\mathtt{abcCar{:}Alice}$ and $\mathtt{abcCar}{:}\mathsf{Customer}$ with $\mathcal{W}_{\mathtt{abcCar}}$.

Definition 9 (Partially Transparent Ontology Space). *Let I be a set of ontology namespace identifiers. A* partially transparent ontology space *(or simply* PTO space*) on I is a family of PTOs $\mathfrak{S}_I = \{\mathcal{W}_{\mathtt{i}} = \langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}}, \mathcal{S}_{\mathtt{i}} \rangle\}_{\mathtt{i} \in I}$ such that $\mathcal{M}_{\mathtt{i}} \subseteq I$ and $[\mathcal{S}_{\mathtt{i}}] \subseteq I$, for each $\mathtt{i} \in I$, and each* $\mathtt{j}$*-foreign URIref $\mathtt{j}{:}x$ occurring in $\mathcal{O}_{\mathtt{i}}$ and $\mathcal{S}_{\mathtt{i}}$ is contained in the local language of $\mathcal{W}_{\mathtt{j}}$.* $\diamond$

Definition 10 (Partially Transparent Distributed Interpretation). *Let I be a set of ontology namespace identifiers and $\mathfrak{S}_I = \{\mathcal{W}_{\mathtt{i}} = \langle \mathcal{M}_{\mathtt{i}}, \mathcal{O}_{\mathtt{i}}, \mathcal{S}_{\mathtt{i}} \rangle\}_{\mathtt{i} \in I}$ a transparent ontology space. A distributed interpretation $\hat{\mathcal{I}} = \{\mathcal{I}_{\mathtt{i}}\}_{\mathtt{i} \in I}$ is called a* Partially transparent distributed interpretation *(or simply* PTD interpretation*) of $\mathfrak{S}_I$ iff for each $\mathcal{W}_{\mathtt{i}} \in \mathfrak{S}_I$ and each shared ontology $\mathcal{W}_{\mathtt{j}}$ of $\mathcal{W}_{\mathtt{i}}$, the following conditions hold:*

- *if* $\mathtt{j}{:}\mathsf{C} \in \mathbf{N}_{\mathtt{j}}$, *then* $(\mathtt{j}{:}\mathsf{C})^{\mathcal{I}_{\mathtt{i}}} = (\mathtt{j}{:}\mathsf{C})^{\mathcal{I}_{\mathtt{j}}} \cap \Delta^{\mathcal{I}_{\mathtt{i}}}$;
- *if* $\mathtt{j}{:}r \in \mathbf{N}_{\mathtt{j}}$, *then* $(\mathtt{j}{:}r)^{\mathcal{I}_{\mathtt{i}}} = (\mathtt{j}{:}r)^{\mathcal{I}_{\mathtt{j}}} \cap (\Delta^{\mathcal{I}_{\mathtt{i}}} \times \Delta^{\mathcal{I}_{\mathtt{i}}})$;
- *if* $\mathtt{j}{:}\mathsf{a} \in \mathbf{N}_{\mathtt{j}}$, *then* $(\mathtt{j}{:}\mathsf{a})^{\mathcal{I}_{\mathtt{i}}} = (\mathtt{j}{:}\mathsf{a})^{\mathcal{I}_{\mathtt{j}}} \in \Delta^{\mathcal{I}_{\mathtt{i}}}$. $\diamond$

Now we revisit Example 4 to illustrate the PTD interpretation defined above. Let $I = \{\mathtt{abcCar}, \mathtt{xyz}\}$ and $\mathfrak{S}_I = \{\mathcal{W}_{\mathtt{abcCar}}, \mathcal{W}_{\mathtt{xyz}}\}$. Let $\mathcal{I}_1 = \langle \Delta^{\mathcal{I}_1}, \cdot^{\mathcal{I}_1} \rangle$ be an interpretation where $\Delta^{\mathcal{I}_1} = \{a, c, j\}$ and the interpretation function $\cdot^{\mathcal{I}_1}$ is defined as follows: $(\mathtt{abcCar}{:}\,\mathsf{Customer})^{\mathcal{I}_1} = \{a, j\}, (\mathtt{abcCar}{:}\,\mathit{order})^{\mathcal{I}_1} = \{\langle a, c\rangle\}, (\mathtt{abcCar}{:}\,\mathtt{Alice})^{\mathcal{I}_1} = a, (\mathtt{abcCar{:}car1})^{\mathcal{I}_1} = c, (\mathtt{abcCar{:}John})^{\mathcal{I}_1} = j$. It is obvious that $\mathcal{I}_1$ is a local interpretation of $\mathcal{W}_{\mathtt{abcCar}}$. Let $\mathcal{I}_2 = \langle \Delta^{\mathcal{I}_2}, \cdot^{\mathcal{I}_2} \rangle$ be an interpretation where $\Delta^{\mathcal{I}_2} = \{a, d\}$ and the interpretation function $\cdot^{\mathcal{I}_2}$ is defined as follows: $(\mathtt{abcCar}{:}\,\mathsf{Customer})^{\mathcal{I}_2} = \{a, d\}, (\mathtt{abcCar{:}Alice})^{\mathcal{I}_2} = a, (\mathtt{xyz{:}David})^{\mathcal{I}_2} = d$. It is obvious that $\mathcal{I}_2$ is a local interpretation of $\mathcal{W}_{\mathtt{xyz}}$. However, $\langle \mathcal{I}_1, \mathcal{I}_2 \rangle$ is *not* a PTD interpretation of $\mathfrak{S}_I$ because

$$(\mathtt{abcCar}{:}\,\mathsf{Customer})^{\mathcal{I}_2} \neq (\mathtt{abcCar}{:}\,\mathsf{Customer})^{\mathcal{I}_1} \cap \Delta^{\mathcal{I}_2}.$$

PTO interpretation must share the meaning (i.e., the interpretation of the element that belongs to the shared box). If we extend $\mathcal{I}_1$ to $\mathcal{I}_1'$, where $\Delta^{\mathcal{I}_1'} = \Delta^{\mathcal{I}_1} \cup \{d\}$ and $\cdot^{\mathcal{I}_1'}$ extends $\cdot^{\mathcal{I}_1}$ with $(\mathtt{abc}{:}\,\mathsf{Customer})^{\mathcal{I}_1'} = (\mathtt{abc}{:}\,\mathsf{Customer})^{\mathcal{I}_1} \cup \{d\}$, then we have $\langle \mathcal{I}_1', \mathcal{I}_2 \rangle$ being a PTD interpretation of $\mathfrak{S}_I$.

Definition 11 (Transparent Entailment). *Let $\mathfrak{S}$ be a partially transparent ontology space on I, ϕ a concept axiom, a role axiom or an individual axiom. We say that $\mathfrak{S}$* entails *ϕ in the PTO $\mathcal{W}_{\mathtt{i}}$ ($\mathtt{i} \in I$), written $\mathfrak{S} \models_{\mathtt{i}} \phi$, if, for every PDT interpretation $\mathcal{I}_{\mathtt{i}}$ of $\mathfrak{S}$, $\mathcal{I}_{\mathtt{i}} \models \phi$.* $\diamond$

In the following theorem we characterise what ontological knowledge is "introduced" to $\mathcal{W}_2$, when it declares ontology $\mathcal{W}_1$ in its shared box.

Theorem 2. *Let $\mathfrak{S} = \langle \mathcal{W}_1, \mathcal{W}_2 \rangle$ be a partially transparent ontology space, $\mathtt{1{:}a}$, $\mathtt{1{:}b}$ individual URIrefs, $\mathtt{1}{:}\mathsf{A}$, $\mathtt{1}{:}\mathsf{B}$ concept URIrefs, and $\mathtt{1}{:}r$, $\mathtt{1}{:}s$ role URIrefs. If $\langle 1, N_1 \rangle \in \mathcal{S}_2$, the following statements hold:*

1. *If $\{\mathtt{1}{:}\mathsf{A}, \mathtt{1{:}a}\} \subseteq N_1$, then $\mathcal{W}_1 \models \mathtt{1}{:}\mathsf{A}(\mathtt{1{:}a})$ implies that $\mathfrak{S} \models_2 \mathtt{1}{:}\mathsf{A}(\mathtt{1{:}a})$ (the entailment of an instance-of-concept axiom is transferred).*
2. *If $\{\mathtt{1}{:}r, \mathtt{1{:}a}, \mathtt{1{:}b}\} \subseteq N_1$, then $\mathcal{W}_1 \models \mathtt{1}{:}r(\mathtt{1{:}a}, \mathtt{1{:}b})$ implies that $\mathfrak{S} \models_2 \mathtt{1}{:}r(\mathtt{1{:}a}, \mathtt{1{:}b})$ (the entailment of an instance-of-role axiom is transferred).*
3. *If $\{\mathtt{1}{:}\mathsf{A}, \mathtt{1}{:}\mathsf{B}\} \subseteq N_1$, then $\mathcal{W}_1 \models (\mathtt{1}{:}\mathsf{A}) \sqsubseteq (\mathtt{1}{:}\mathsf{B})$ implies that $\mathfrak{S} \models_2 (\mathtt{1}{:}\mathsf{A}) \sqsubseteq (\mathtt{1}{:}\mathsf{B})$ (the entailment of an atomic concept axiom is transferred).*
4. *If $\{\mathtt{1}{:}r, \mathtt{1}{:}s\} \subseteq N_1$, then $\mathcal{W}_1 \models (\mathtt{1}{:}r) \sqsubseteq (\mathtt{1}{:}s)$ implies that $\mathfrak{S} \models_2 (\mathtt{1}{:}r) \sqsubseteq (\mathtt{1}{:}s)$ (the entailment of an atomic role axiom is transferred).*

Proof: For (1). Let $\langle \mathcal{I}_1, \mathcal{I}_2 \rangle$ be a PTD interpretation for $\mathfrak{S}$. Since $\mathcal{W}_1 \models \mathtt{1}{:}\mathsf{A}(\mathtt{1{:}a})$, we have $(\mathtt{1{:}a})^{\mathcal{I}_1} \in (\mathtt{1}{:}\mathsf{A})^{\mathcal{I}_1}$. Due to Definition 10, $\langle 1, N_1 \rangle \in \mathcal{S}_2$ and $\{\mathtt{1}{:}\mathsf{A}, \mathtt{1{:}a}\} \subseteq N_1$, we have $(\mathtt{1{:}a})^{\mathcal{I}_2} \in \Delta^{\mathcal{I}_2}$. Therefore, we have $(\mathtt{1{:}a})^{\mathcal{I}_2} \in (\mathtt{1}{:}\mathsf{A})^{\mathcal{I}_2}$; hence, $\mathfrak{S} \models_2 \mathtt{1}{:}\mathsf{A}(\mathtt{1{:}a})$.

For (2). Let $\langle \mathcal{I}_1, \mathcal{I}_2 \rangle$ be a PTD interpretation for $\mathfrak{S}$. Since $\mathcal{W}_1 \models \mathtt{1}{:}r(\mathtt{1{:}a}, \mathtt{1{:}b})$, we have $\langle (\mathtt{1{:}a})^{\mathcal{I}_1}, (\mathtt{1{:}b})^{\mathcal{I}_1} \rangle \in (\mathtt{1}{:}r)^{\mathcal{I}_1}$. Due to Definition 10, $\langle 1, N_1 \rangle \in \mathcal{S}_2$ and $\{\mathtt{1}{:}r, \mathtt{1{:}a}, \mathtt{1{:}b}\} \subseteq N_1$, we have $\{(\mathtt{1{:}a})^{\mathcal{I}_2}, (\mathtt{1{:}b})^{\mathcal{I}_2}\} \subseteq \Delta^{\mathcal{I}_2}$. Therefore, we have $\langle (\mathtt{1{:}a})^{\mathcal{I}_2}, (\mathtt{1{:}b})^{\mathcal{I}_2} \rangle \in (\mathtt{1}{:}r)^{\mathcal{I}_2}$; hence, $\mathfrak{S} \models_2 \mathtt{1}{:}r(\mathtt{1{:}a}, \mathtt{1{:}b})$.

For (3). Let $\langle \mathcal{I}_1, \mathcal{I}_2 \rangle$ be a PTD interpretation for $\mathfrak{S}$. Since $\mathcal{W}_1 \models (\mathsf{1}{:}\mathsf{A}) \sqsubseteq (\mathsf{1}{:}\mathsf{B})$, we have $(\mathsf{1}{:}\mathsf{A})^{\mathcal{I}_1} \subseteq (\mathsf{1}{:}\mathsf{B})^{\mathcal{I}_1}$. Due to Definition 10, $\langle 1, N_1 \rangle \in \mathcal{S}_2$ and $\{\mathsf{1}{:}\mathsf{A}, \mathsf{1}{:}\mathsf{B}\} \subseteq N_1$, we have $(\mathsf{1}{:}\mathsf{A})^{\mathcal{I}_2} = (\mathsf{1}{:}\mathsf{A})^{\mathcal{I}_1} \cap \Delta^{\mathcal{I}_2} \subseteq (\mathsf{1}{:}\mathsf{B})^{\mathcal{I}_1} \cap \Delta^{\mathcal{I}_2} = (\mathsf{1}{:}\mathsf{B})^{\mathcal{I}_2}$. Hence, $\mathfrak{S} \models_2 (\mathsf{1}{:}\mathsf{A}) \sqsubseteq (\mathsf{1}{:}\mathsf{B})$. The proof for (4) is similar to that for (3). $\square$

The main message of the theorem is that shared boxes allow ontologies to share some *data* as well as some (explicit or implicit) *axioms* (w.r.t. the shared data) with other ontologies.

5 Related work

A first notion of OWL space was originally defined in [BGvH+03] as a tool whose definition was preliminary to the definition of context mappings. As intense follow-up discussions (which consequently lead to the writing of this paper) have shown, this notion is actually more important than we originally thought. It allows us to provide a semantics of how the OWL importing operator works, and more in general, of the possible interactions between multiple ontologies. As a side result, this paper shows that the semantics of OWL spaces, as described in [BGvH+03] do not capture the intended semantics of multiple ontologies linked by the importing operator. In that paper the authors in fact assumed that the import operator had transparent semantics.

The opaque and transparent semantics for ontology spaces are obtained by modifying the original OWL semantics using the ideas and notions originally developed in the semantics of context (the, so called, Local Models Semantics [GG01] and its extension to first order logic [GS98]). Roughly, the local model semantics for a distributed system (where each component is called context) associates to each context a local interpretation, but only certain combinations of local interpretations are admitted, i.e, the ones that respect a *compatibility condition*. In the opaque semantics, the compatibility condition is always true, as all the combination of local interpretations for each ontology is admitted. In partially transparent distributed interpretations, instead, the local interpretations of the ontologies with a non empty shared box $\mathcal{S}_\mathtt{i}$ have to respect the compatibility conditions 1, 2, and 3 of Definition 10.

The notion of shared box with transparent interpretation, as proposed in this paper, is orthogonal with respect to the C-OWL language, and one could easily imagine a situation in which C-OWL and shared boxes are combined in a unique framework. From a semantic point of view, here we have adopted a semantics which is homogeneous with respect to the C-OWL semantic (they are both based on LMS [GG01,GS98]), and therefore they can be easily integrated to provide a semantics of the global framework. In particular, the main contribution of C-OWL is a set of primitives which allow to state semantic similarity, between concept, roles, and individuals of different ontologies. Domains are supposed to be distinct and a domain relation allow to translates objects of a domains into object in another domain. Transparent semantics, instead, allows to *share* object in a domain, i.e., to allow co-reference to the same "real world" object from two different ontologies.

The intuition underlying the notion of shared box has a lot of commonalities with the notion of *rigid interpretation/rigid designation* in quantified modal logics. Roughly if an individual constant $\mathtt{i{:}c}$ belongs to a shared box of an ontology $\mathcal{W}_\mathtt{j}$, it interpretation in $\mathcal{I}_\mathtt{i}$ and $\mathcal{I}_\mathtt{j}$ is rigid, i.e,, it is interpreted in the same object. As a further investigation on this analogy we plan to reuse some of the theoretical results developed in quantified modal logics to prove properties on shared meaning in the Semantic Web.

6 Conclusion

The main contribution of this paper is the notion of OWL space. OWL spaces allow us to model the fact that, once published, ontologies can be used and reused by many other ontologies and users, with possibly very different intended meanings. The notion of opaque interpretation, which, among other things, formalizes the intended behavior of the OWL importing mechanism, captures the fact that each ontology in a OWL space has its own semantics, independently of the surrounding ontologies. The notion of transparent interpretation allows us to finely tune, on a predefined set of language elements, the semantics of one ontology with respect to the semantics of other reference ontologies.

This work is part of a much bigger effort which aims at studying the possible interactions of ontologies in an ontology space. Much more work needs to be done. For instance: we need to provide a syntax which allows to state that we want a transparent interpretation on a certain language element, we need to integrate this work with the work on C-OWL and provide a uniform syntax for all these new constructs; we need to start worrying about the interactions among sub-ontologies, and so on.

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