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Abstract

Several studies have shown that evolutionary-based approaches are efficient, effective, and robust optimization methods for microwave imaging. However, the convergence rate of such techniques still does not meet all the requirements for on-line real applications and attempting to speed up the optimization is needed. In this paper, a new local search operator, the fitness-based parabolic crossover, is proposed and embedded into a real-coded genetic algorithm. Such a modification enables the imaging method to achieve a better trade-off between convergence rate and robustness to false solutions. By exploiting the relationship between the crossover operation and the local quadratic behavior of the functional, it is possible to increase the convergence rate of the genetic algorithm and thereby to obtain an acceptable solution with a smaller number of fitness function evaluations. The effectiveness of the modified genetic-algorithm-based imaging method is assessed by considering some synthetic test cases different in dimensions and noisy conditions. The obtained numerical results provide an empirical evidence of the efficiency and reliability of the proposed modified evolutionary algorithm.

Index Terms - Microwave imaging, inverse scattering, genetic algorithms, evolutionary operators

1 Introduction

Evolutionary algorithms (EAs) are a class of nonlinear optimization approaches based on the natural selection and on the principle of the "survival of the fittest". The robustness, the flexibility, the intrinsic parallelism, and the global search capability are typical features, which make EAs applicable and very attractive within a large framework of engineering disciplines. The Genetic Algorithm (GA) developed by Holland [1] and deeply studied by Goldberg [2] some years ago, is one of the most promising and diffuse EAs. Such an approach has been demonstrated to be an efficient, effective, and robust optimization method in several area and in particular for microwave imaging applications [3][4][5][6][7]. Since GAs (as well as all EAs) are *multiple-agent* techniques, they work with a population of solutions rather than a single solution and a high number of trial solutions are evaluated during the optimization. Evaluating a candidate solution is not very difficult, but unfortunately it is time-consuming and, in some cases, the overall computational burden needed to find the global optimum could be unacceptable. Therefore, it is obvious that further studies on suitable strategies able to "locally" speed up the convergence are still an interesting and important topic. As a matter of fact, such an acceleration may allow better solutions to be obtained within the limited amount of CPU-time available for optimization in industrial processes.

As far as the reduction of the computational burden of GAs is concerned, different strategies can be adopted. Taking into account that the CPU-time, needed to reach a feasible solution of the problem at hand, is due to the time spent at each iteration and to the total number of iterations, suitable countermeasures can be adopted. Neglecting the reduction of the CPU-time for one iteration (addressed in [8] through a parallel implementation), a possibility is to implement a suitable modification of the searching procedure to reduce the total number of iterations. Such a result can be achieved by defining suitable genetic operators able to construct more promising offspring on the basis of two or more parents, then allowing an improvement in the convergence towards the global optimum. Let us consider the crossover. The construction of a crossover operator depends on different aspects: the coding procedure and the implicit hypothesis on such a function around the selected parents.

By considering a real-coding, many effective crossover operators have been proposed (see [9] for an overview) ranging from simplest methods, which choose one or more points in the coded-solution (i.e., the *chromosome*) to mark the crossover points, to the *arithmetical crossover* [10], where the offspring are a linear combination of their parents, and the *geometrical crossover* [11]. On the other hand, by not assuming a linear but a parabolic model of the cost function surface, Stidsen *et al.* proposed in [12] a new recombination operator that suitably links the crossover to the *a-priori* knowledge on the behavior of the cost function.

In the same framework, this paper is aimed at presenting a new GA method based on a fitness-based crossover (GAPC) to fully exploit some *a-priori* knowledge on the local behavior of cost function in hand. It is well known that in nonlinear inverse scattering problems the cost function presents several minima. This is certainly a drawback, but such an information can be conveniently exploited in designing an evolutionary operator. A minimum defines an attraction basin where certainly the cost function shows a quadratic behavior. Therefore, even though the cost function is not exactly known, a quadratic model can be assumed as a good approximation of the local behavior of the function.

The paper is organized as follows. In Section 2, the mathematical model of an inverse scattering problem is briefly described by defining the nonlinear optimization problem to be solved. With reference to a known real-coded version of the genetic algorithm, the evolution strategy based on the innovative fitness-based crossover is presented in Section 3. Selected results from numerical experiments are presented in Section 4 to validate the proposed approach. Some brief conclusions follows (Sect. 5).

2 The Mathematical Model

In many applications, an optimization procedure searching for the global minimum of a suitably-defined cost function is the leading way to solve the problem in hand. Concerning microwave imaging, optimization algorithms demonstrated to work properly. Generally speaking, a microwave imaging problem formulated in terms of inverse scattering equations involves the retrieval of an unknown scenario illuminated by a set of V incident fields (E_I^v, $v = 1, ..., V$) staring from the knowledge of the scattered field E_S^v measured in a region external to the investigation domain D_{ind} where the scatterer is supposed to be located. According to the well-known inverse scattering equations, the problem can be mathematically described as follows [4]:

$$E_S^v = G_2^v \tau E_T^v \quad (Data\ Equation) \tag{1}$$

$$E_I^v = E_T^v - G_1^v \tau E_T^v \quad (State\ Equation) \tag{2}$$

where G_1^v and G_2^v are the internal and external Green's functions [13], respectively; τ is the unknown object function describing the dielectric distribution in D_{ind} and E_T^v is the electric field in the investigation domain concerning the v-th illumination.

Because of the ill-conditioning and the nonlinearity of the problem, caused by the multiple scattering effects and the limited amount of information of the data, the solution of the problem is cast as the minimization of the following cost function:

$$F\left(\tau,\, E_T\right) = \frac{\sum_{v=1}^{V} \left\| E_S^v - G_{ext} \tau E_T^v \right\|^2}{\sum_{v=1}^{V} \left\| E_S^v \right\|} + \frac{\sum_{v=1}^{V} \left\| E_I^v - E_T^v + G_{int} \tau E_T^v \right\|^2}{\sum_{v=1}^{V} \left\| E_I^v \right\|^2} \tag{3}$$

characterized by a large number of local minima corresponding to false solutions of the inverse problem. To minimize such a function, a global optimization procedure able to avoid local minima is generally used. A technique widely used is based on a GA [1].

3 The Optimization Procedure

With reference to the Real-Coded Genetic Algorithm (RGA) presented in [4], let us consider the chromosome $\underline{x}$ which is the floating point representation of the problem unknowns ($\underline{x} = \{\tau, E_T^v; v = 1, ..., V\}$). The optimizer works iteratively on a set of chromosomes called *population*, $P_k = \left\{\underline{x}_k^{(l)}; l = 1, ..., L\right\}$, L being the dimension of the population and k the iteration index. According to an evolutionary strategy, every solution is ranked according to its fitness value, $F_k^{(l)} = F\left(\underline{x}_k^{(l)}\right)$, and a new population P_{k+1} is determined through the application of the genetic operators. Their computational role is to introduce diversity into the population, by probing new regions unexplored by the selection operator [14]. The mutation operator inserts variability in the population by modifying the value of one or more positions (or *genes*) in a selected chromosome. The crossover performs parents' reproduction and it generates a new individual $\underline{x}_{k+1}^{(l)}$ by crossing two selected parents of the current population. Standard floating-point crossover operators as well as that used in [4] produce an offspring $\underline{x}_{k+1}^{(l)}$ as the linear combination of its parents $\underline{x}_k^{(i)}$ and $\underline{x}_k^{(j)}$ $(i, j \in [1, L]; i \neq j)$

$$\underline{x}_{k+1}^{(l)} = t\underline{x}_k^{(i)} + (1-t)\underline{x}_k^{(j)} \tag{4}$$

t being a random real number. In order to show the behavior of such an arithmetical operation, let us consider the simple mono-dimensional sketch shown in Fig. 1. As can be inferred, the operator does not consider the shape of the function to define the offspring and the new solution could lie in another attraction basin, even though its parents belong to that of the global minimum. This event certainly slows the convergence rate of the minimization process by increasing the computational cost and the overall CPU-time.

To limit such a drawback, a new version of the floating-point crossover for inverse scattering problems has been designed by taking advantage of the locally-quadratic behavior of the cost function. Referring to Fig. 2, let us define the trial chromosome $\underline{x}_{t_r} = t\underline{x}_k^{(i)} + (1-t)\underline{x}_k^{(j)}$, $t = t_r$ between $\underline{x}_k^{(i)}$ and $\underline{x}_k^{(j)}$ by choosing the random value t_r in the range $0 \leq t_r \leq 1$ and satisfying the following relationship

$$F\left(\underline{x}_{t_r}\right) < \left\{F\left(\underline{x}_k^{(j)}\right) - F\left(\underline{x}_k^{(i)}\right)\right\} t_r + F\left(\underline{x}_k^{(i)}\right) \tag{5}$$

Then the new chromosome is obtained as the vertex of the parabola passing through $\underline{x}_k^{(i)}$, $\underline{x}_k^{(j)}$, and $\underline{x}_{t_r}$

$$f(t) = at^2 + bt + c \tag{6}$$

where $a = \frac{(1-t_r)F\left(\underline{x}_k^{(i)}\right)+t_r \cdot F\left(\underline{x}_k^{(j)}\right)-F\left(\underline{x}_{t_r}\right)}{t_r(1-t_r)}$, $b = \frac{F\left(\underline{x}_{t_r}\right)-F\left(\underline{x}_k^{(i)}\right)+t_r^2\left\{F\left(\underline{x}_k^{(i)}\right)-\left(\underline{x}_k^{(j)}\right)\right\}}{t_r(1-t_r)}$, and $c = F\left(\underline{x}_k^{(i)}\right)$. Accordingly, the offspring turns out to be

$$\underline{x}_{k+1}^{(l)} = t_v \underline{x}_k^{(i)} + (1-t_v)\underline{x}_k^{(j)}, \quad t_v = -\frac{b}{2a} \tag{7}$$

The proposed "parabolic crossover" similarly to that proposed in [12] acts as both exploration and exploitation operator, and is a multi-parent operator. However, the number of parents does not depend on the dimensionality of the problem. Only three parents are needed to determine an offspring and not $O\left(P^2\right)$ (P being the number of unknowns or dimensionality proportional to the problem size $N^{(1)}$ through such a relationship $P = 2 \times N \times (V+1)$), since the new recombination operator acts in the mono-dimensional solution space (sampled by t) and not in the unknowns space (whose dimension is P).

4 Numerical Validation

In this section, selected numerical results from a large computational assessment are shown to give some indications on the effectiveness of the improved GA-based procedure. As a reference test case, a synthetic two-dimensional scenario has been considered consisting of a homogeneous ($\tau = \tau(x, y) = 1.5$) square cylinder $0.225\,\lambda$-sided centered at $x_c = y_c = 0.3\,\lambda$ in an investigations domain of side $L_{D_{ind}} = 1.125\,\lambda$. The unknown object is illuminated by a set of $V = 4$ TM-polarized plane waves and the scattered field is collected

(1) According to the Richmond's procedure [16], Eqs. (1)-(2) have been discretized by considering N equal partitions of D_{ind} where both the dielectric characteristics and the induced field are assumed to be constant.

by means of a multi-illumination/multi-view system [15] in $M = 10$ measurement points located on a circle $R = 1.125\,\lambda$ in radius. According to the Richmond's procedure [16], the investigation domain has been partitioned in $N = 16 \times 16$ equal subdomains, thus $P = 2048$.

As far as the GAs parameters are concerned, they have been chosen according to the values suggested in the reference literature on this subject [17][14]: $L = 200$, $P_c = 0.8$ (crossover probability), $P_m = 5\times10^{-2}$ (mutation probability), $P_{bm} = 1\times10^{-3}$ (single-gene mutation), and $K = 2 \times 10^4$ (maximum number of iterations). The initial population P_0 has been randomly generated around the free-space trial solution ($\underline{x}_0^{(1)} = \{\tau_0,\, E_I^v;\, v = 1, ..., V\}$, $\tau_0 = 0.0$) in the following ranges defined according to the available *a-priori* information on the scenario under test: $0.0 \leq Re\left\{\tau(x,\,y)\right\} \leq 2.0$, $-1.5 \leq Re\left\{E_T^v(x,\,y)\right\} \leq 1.5$, and $-1.5 \leq Im\left\{E_T^v(x,\,y)\right\} \leq 1.5$.

For comparison purposes, to point out the effectiveness of the improved approach in minimizing (3), Fig. 3 shows the behavior of the fitness of the optimal solution (i.e., $F_k^{(opt)} = min_{h=1,...,k}\left\{min_l\left[F\left(\underline{x}_h^{(l)}\right)\right]\right\}$) during the iterative process. As can be noticed, at the end of the process, the optimal value of the cost function is reduced of about 4 orders in magnitude when the GAPC is used, while the decreasing is about of only two orders with the RGA-based method.

Such a behavior causes an improvement in the reconstruction accuracy of the scenario under test as quantitatively confirmed by the values of the error figures defined as follows

$$\Xi_j = \left\{\int_{D_j} \frac{\tau^{(opt)}(x,y) - \tilde{\tau}(x,y)}{\tilde{\tau}(x,y)} dxdy\right\} \times 100 \qquad (8)$$

$\tau^{(opt)}$ and $\tilde{\tau}$ are the retrieved and the actual object function, respectively; D_j indicates the whole investigations domain ($j \Longrightarrow tot$), or the area where the actual scatterer is located ($j \Longrightarrow int$), or the background belonging to the investigation domain ($j \Longrightarrow ext$). More in detail, the average values of 10 repeated independent realizations of the imaging process turn out to be $\Xi_{tot}\rfloor^{(GAPC)} = 0.60$ vs. $\Xi_{tot}\rfloor^{(RGA)} = 3.93$, $\Xi_{int}\rfloor^{(GAPC)} = 11.45$ vs.

$\Xi_{int}\rfloor^{(RGA)} = 36.07$, and $\Xi_{ext}\rfloor^{(GAPC)} = 0.15$ vs. $\Xi_{ext}\rfloor^{(RGA)} = 2.59$.

Moreover, to assess also the robustness of the new approach against the noise, different noisy conditions have been taken into account by adding to the inverse data a gaussian noise characterized by various signal-to-noise ratios (SNRs). Figure 4 shows the plots of Ξ_j versus the SNR. Once again, the values of the error figures achieved with the GAPC are lower than those of the RGA.

From a computational point of view, the parabolic crossover is expected to be more time consuming than the standard arithmetic crossover [4] then it could be interesting to evaluate the trade-off between the improvement in the reconstruction accuracy and the increment of the overall computational load. Towards this end, an exhaustive computational analysis has been carried out by considering different sizes of the problem ranging from $N = 2 \times 2$ to $N = 16 \times 16$. As expected, the CPU-time needed to complete an iteration increases with N and the GAPC is slightly more expensive that the RGA [Fig. 5(a)], but the parabolic crossover, thanks to the local search capabilities, requires a smaller number of iterations to achieve the convergence [Fig. 5(b)]. Such a behavior is also confirmed by varying the SNR as shown in Fig. 6 dealing with a dimensionality of $P = 2048$.

5 Conclusions

A new recombination strategy, the parabolic crossover has been proposed and integrated into a real-coded genetic algorithm for microwave imaging purposes. Since the parabolic crossover performs a local sampling of the solution space in a global hill-climbing search, it allows to implicitly balance the convergence rate and robustness against local minima or false solutions.

The improved strategy has been assessed by considering noiseless as well noisy data and various problem dimensions. Moreover, a comparative study has been carried out to evaluate the improvement over the original approach. The numerical results have demonstrated that the reconstruction accuracy of the retrieval process could be enhanced as well as the convergence rate of the optimization.

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Figure Captions

- **Figure 1.** Example of the behavior of the fitness for the individuals involved in a standard RGA-crossover operation.

- **Figure 2.** Pictorial description of the action of the parabolic crossover.

- **Figure 3.** *Analysis of the Optimizer Effectiveness* - Behavior of cost function versus the iteration number k.

- **Figure 4.** *Analysis of the Reconstruction Capabilities* - Behavior of the error figures versus SNR: (a) Ξ_{tot}, (b) Ξ_{int}, and (c) Ξ_{ext}.

- **Figure 5.** *Analysis of the Computational Burden* - Comparison between the RGA-based approach and the GAPC-procedure: (a) normalized CPU iteration time and (b) percentage of iterations needed to achieve a fixed threshold $F_k^{(opt)} < \eta$, $\eta = 10^{-2}$.

- **Figure 6.** *Analysis of the Computational Burden* - Percentage of iterations needed to reach the convergence versus SNR by using the RGA-based procedure and GAPC-based approach.

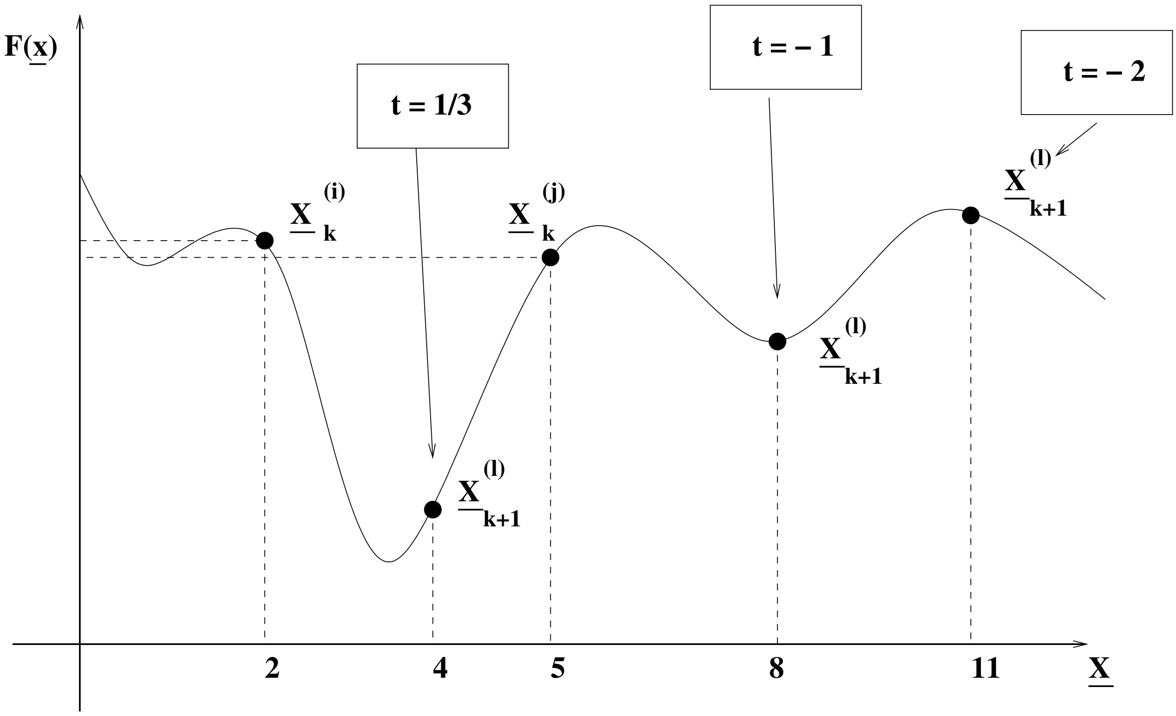
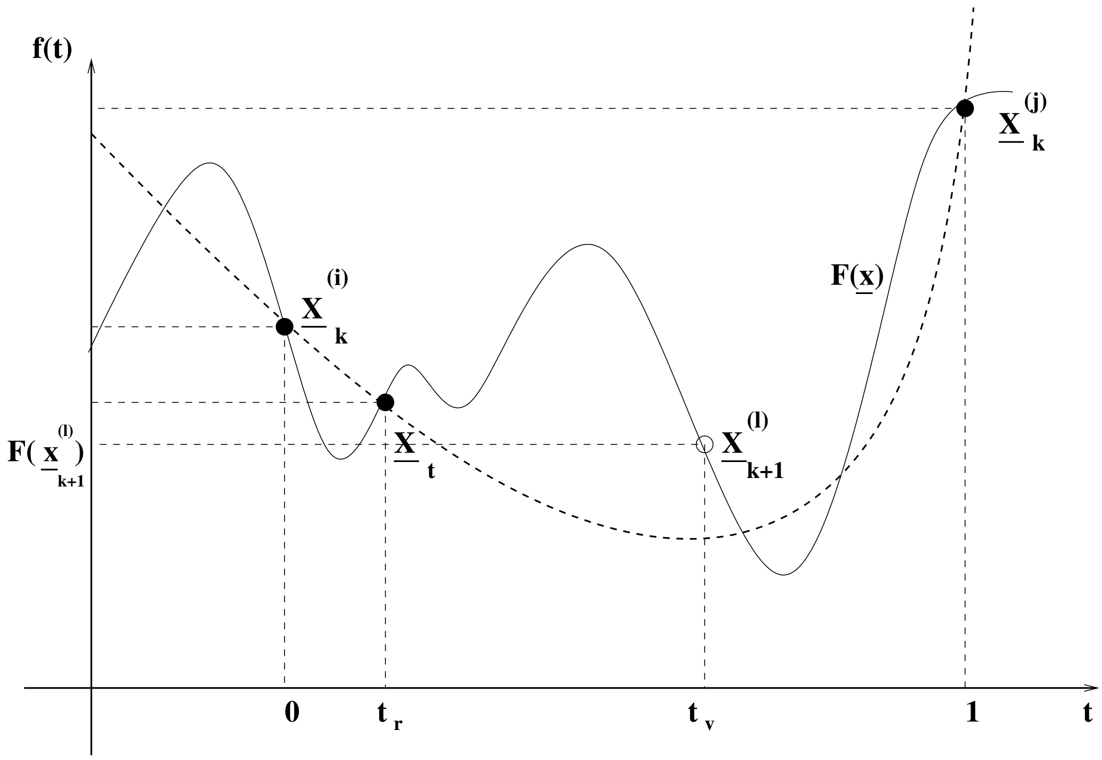


Figure 1 - E. Bort *et al.*, "Improving the Effectiveness of ..."



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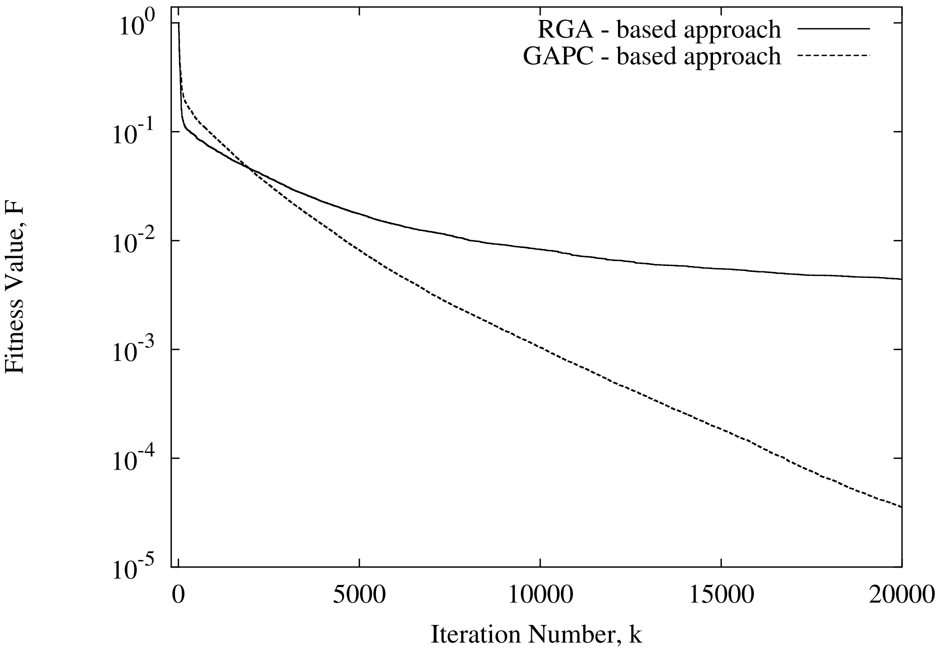

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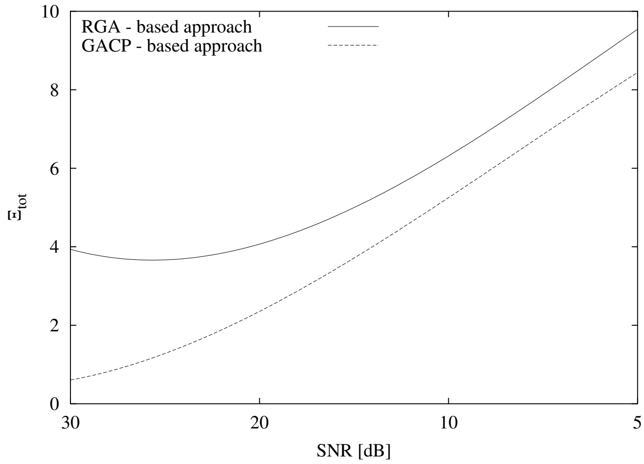
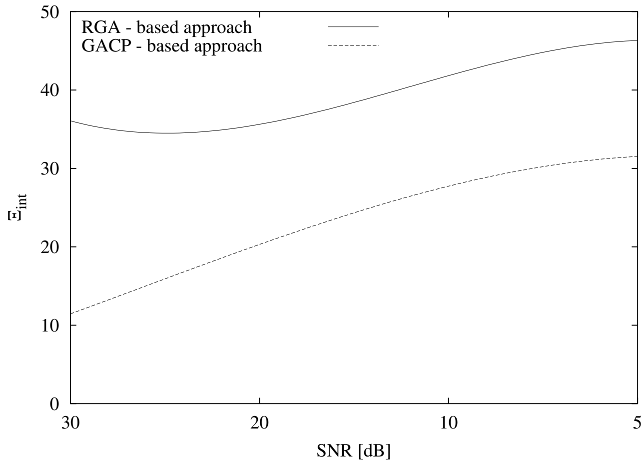


(a)



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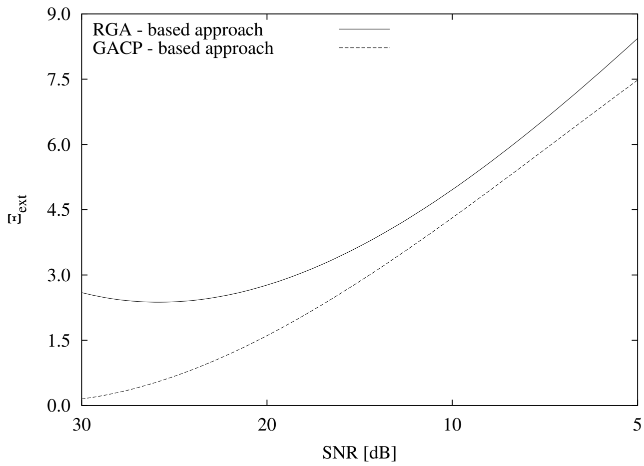
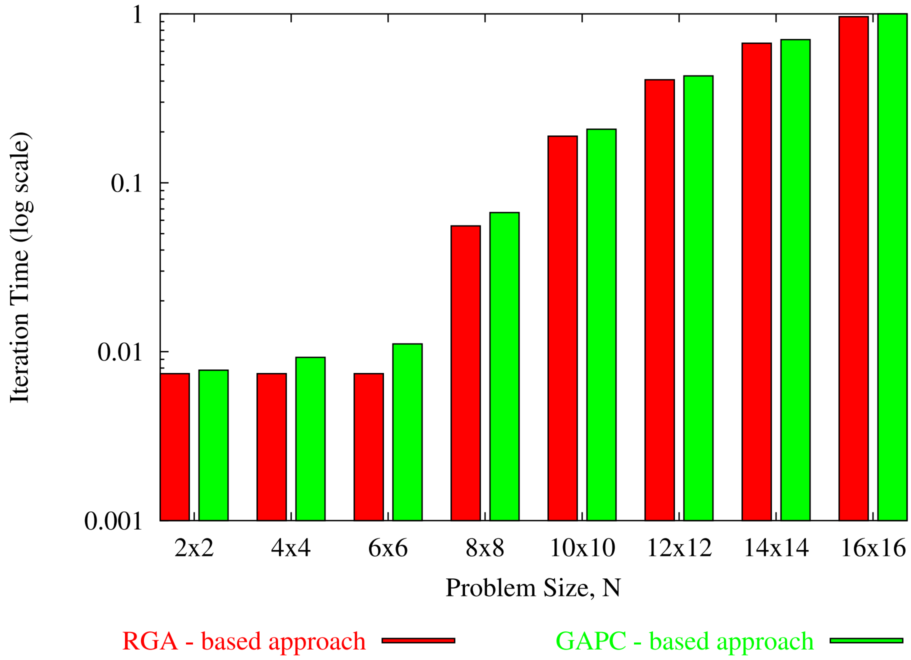
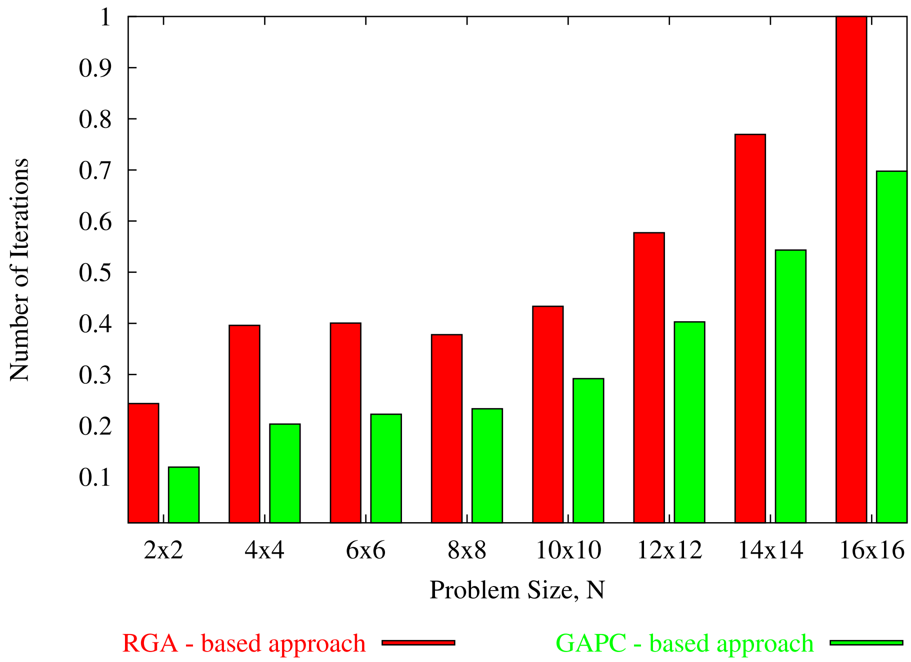


(c)

Figure 4 - E. Bort *et al.*, "Improving the Effectiveness of ..."



(a)



(b)

Figure 5 - E. Bort *et al.*, "Improving the Effectiveness of ..."

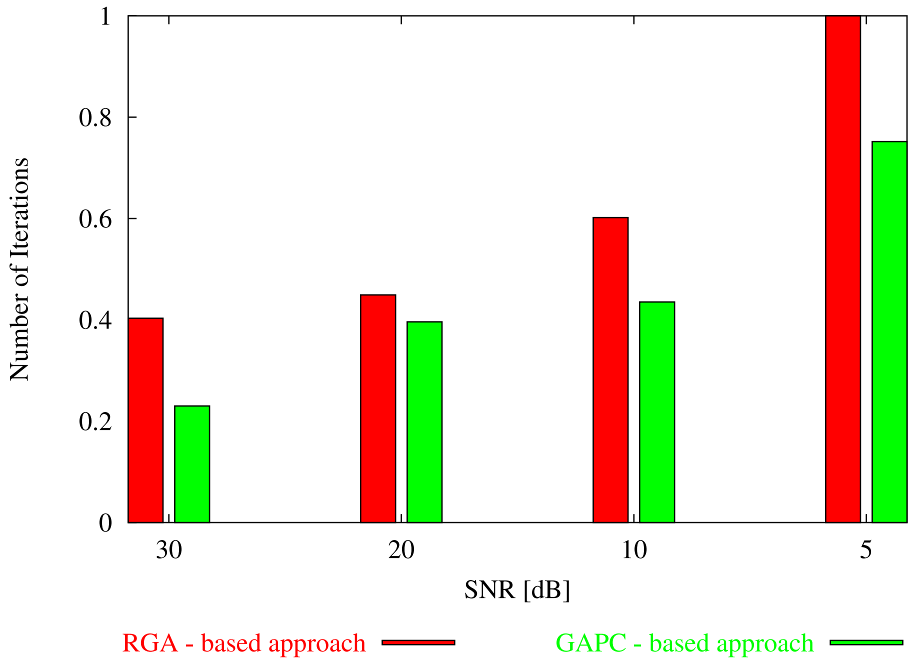


Figure 6 - E. Bort *et al.*, "Improving the Effectiveness of ..."