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Measurements of Transient Phenomena With Digital Oscilloscopes

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Abstract ***–In this paper, the effects of sampling upon rise time measurements with a digital oscilloscope are considered. In particular, the use of linear interpolation for estimating signal rise times is discussed, and its effects are analyzed for various step signals. A simple expression is derived, which accurately models the sampling and linear interpolation contributions to the overall rise time measurement error. Using these results, a correction formula is proposed, and its applicability is discussed.***

I. INTRODUCTION

Digital Storage Oscilloscopes (DSOs) are nowadays widely used in many measurement processes, often replacing analog ones. Such devices, after conditioning and A/D converting the input signal, perform most measuring processing into the digital domain, thus offering high and reproducible performances. In order to evaluate the uncertainty associated to measurements of wideband signals or transient phenomena, it is very important to characterize the dynamic performance of both analog and digital oscilloscopes. To this aim, it has been shown that the knowledge of analog circuitry bandwidth and the use of some empirical rules usually applied to estimate its effect on a transition time measurement are not fully justified [1].

Moreover, when a DSO is employed, the threshold crossing times of a signal are often estimated by means of linear interpolation. Thus, measurements of transient phenomena are affected by a further source of error, which, to the best of the knowledge of the authors, has not been adequately addressed yet.

In this paper, the effects of sampling on the rise time measurements are analyzed, for various step signals. In this regard, it should be noticed that the sampled signal is not the DSO input signal, but the output of the analog circuitry of the instrument [1],[2],[3]. First, simple step signal analytical models are considered. Then more realistic waveforms are achieved as the output of a second-order system fed by simple step signals [1]. In both cases, the error introduced on the rise time measurement by sampling and linear interpolation is evaluated by means of simulations. A formula is then derived, and its usefulness in compensating measurement error for a wide class of input signals is discussed.

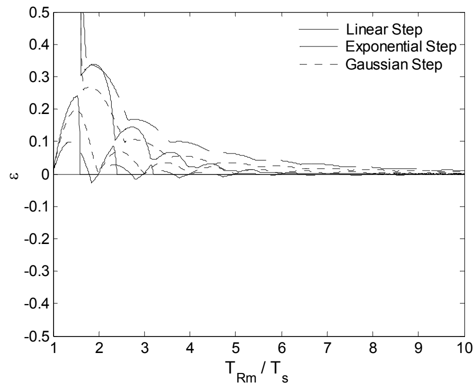

Fig. 1: Relative error ε due to sampling and linear interpolation on rise time measurements

II. EFFECT OF SAMPLING AND LINEAR INTERPOLATION

The rise time of a step signal is usually defined as the difference between the instants $t_{10\%}$ and $t_{90\%}$ in which the signal crosses two threshold values, conventionally assumed as 10% and 90% of the step amplitude respectively. A DSO measures the rising time of a step signal by evaluating the difference between the estimated values for $t_{10\%}$ and $t_{90\%}$. To this aim, the instrument often performs a linear interpolation between the last collected sample lower than the reference threshold level and the following sample. This process introduces an estimation error, which depends also on the delay between the signal transition and the sampling instants. At first, three signals are kept into account: a linear, an exponential, and a Gaussian step signal respectively. The linear and exponential steps are often used to model physical phenomena, while the Gaussian one is often adopted to describe the overall step response of a set of cascaded linear filters and amplifiers [1]. Each step signal has been normalized, thus presenting a unitary ideal rise time and unitary amplitude. The relative error $\varepsilon=(T_{Rm}-T_R)/T_{Rm}$, where T_R is the signal rise time and T_{Rm} is the measured rise time, is evaluated by means of both meaningful simulations and theoretical analysis, which is reported in the appendix.

Fig. 1, obtained for various step signals, reports ε as a function of the ratio between T_{Rm} and the sampling period T_S. For each considered waveform, two curves are plotted, which represents the maximum error ε_{max} and the minimum error ε_{min} respectively, achieved by varying the delay between the signal transition and the sampling instants. When the T_{Rm}/T_S ratio is low, the error grows quickly, because the measured time tends to the sampling period. For higher values of T_{Rm}/T_S, the error magnitude tends to zero due to the improved resolution in estimating $t_{10\%}$ and $t_{90\%}$, and can be upper bounded with a hyperbolic law (see the appendix). In particular, Fig. 1 shows that in order for ε_{max} to be below 10%, T_{Rm}/T_S should exceed 4. Moreover, it can be observed that, for the exponential step, a low T_{Rm}/T_S ratio may lead to a negative error.

It is also worth noticing that when T_{Rm} is an integer multiple of T_S, the error can be null. Such a behavior can be explained by considering that in such a situation two samples may exist, which equal the 10% and 90% step reference levels. Consequently, the corresponding sampling times are exactly $t_{10\%}$ and $t_{90\%}$, and no interpolation error is introduced. Furthermore, when the measured signal is a linear step, sampling does not introduce an estimation error when T_{Rm}/T_S equals or exceeds 8. In fact, under such constraint, it can be easily shown that the interpolating straight lines exactly reproduce the linear step rising front.

III. ERROR ESTIMATION AND CORRECTION

The measured rise time T_{Ro} of an analog oscilloscope is usually expressed as follows

$$T_{Ro} = \sqrt{T_R^2 + T_O^2}, \qquad (1)$$

where T_O is the oscilloscope rise time [1]-[5]. Fig. 1 suggests that a similar expression can be adopted for the mean value of ε, thus expressing the measured rise time as

$$T_{Rm} = \sqrt{T_R^2 + \alpha^2 T_S^2}, \qquad (2)$$

where α is a suitable coefficient used to model the effects of sampling and linear interpolation. By applying (2) and the definition of ε, the following expression can be derived,

$$\varepsilon_{\text{int}} = 1 - \sqrt{1 - \alpha^2 T_S^2 / T_{Rm}^2}, \qquad (3)$$

where ε_{int} models the mean value of ε as a function of the T_{Rm}/T_S ratio. Thus, by properly fitting (3) to simulation results, it is possible to evaluate the coefficient α, which allows to estimate the correction term in (2).

Fig. 2 reports the curves ε_{int}, obtained by fitting (3) to the mean value of ε, with the least squares method, for the three considered input signals. The related values of α provided by the algorithm for $1 \le T_{Rm}/T_S \le 10$ are reported in tab.1.

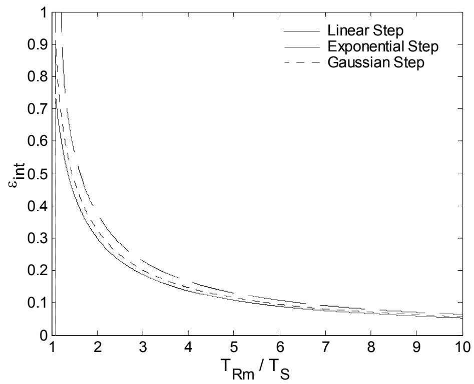

Fig. 2: Interpolating functions for the average error ε_{avg}, obtained for the considered input step signals

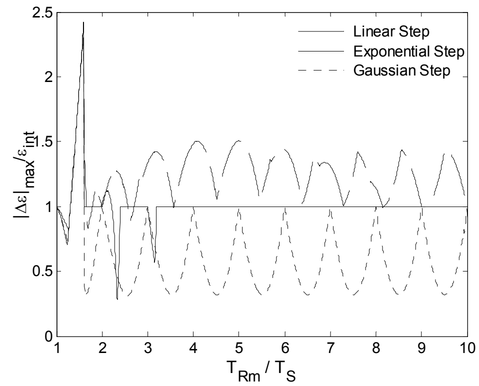

Fig. 3: Envelope of the absolute value of the difference between the relative error ε and the interpolated error ε_{int}, normalized to ε_{int}.

Linear Step	α=1.02
Exponential Step	α=1.22
Gaussian Step	α=1.09

Tab. 1: Fitting parameter α, evaluated for various input step functions and $1 \le T_{Rm}/T_S \le 10$

It can be observed that the interpolating curves of Fig. 2 are very close to each other, which suggests that sampling effects are weakly affected by the shape of the step signal, at least when waveforms with negligible ripple are considered. Such a result is validated by theoretical analysis, which provides values of α quite close to the ones obtained from simulations.

Thus, the bias of the rise time measurement error introduced by sampling and linear interpolation may be corrected by using the following relationship:

$$T'_{Rm} = \sqrt{T_{Rm}^2 - \alpha^2 T_S^2}, \qquad 1 \le \frac{T_{Rm}}{T_S} \le 10, \tag{4}$$

where T'_{Rm} is the corrected rise time and the coefficient α is assumed independent of the shape of the step signal. In particular, the results described in the following have been obtained by choosing α=1.1.

In order to gain better insights on the correction effectiveness, the error properties have been further analyzed. Fig. 3 shows the maximum magnitude of the difference $\Delta\varepsilon=\varepsilon-\varepsilon_{int}$, normalized to ε_{int}, as a function of T_{Rm}/T_S, for the considered step signals. In all of the considered cases, ε_{int} is comparable to the maximum magnitude of the error. Hence, it is expected that removing the bias may be an effective way to improve the measurement accuracy. To gain a further insight, the residual error $\varepsilon'=(T'_{Rm}-T_R)/T'_{Rm}$, obtained by applying (4), has been evaluated and reported in Fig. 4 as a function of T_{Rm}/T_S. For each considered waveform both the maximum and minimum error curves are reported. As expected, Fig. 4 shows that the error mean is negligible. However, outside the interval $2 \le T_{Rm}/T_S \le 10$, the use of (4) is not advantageous. In fact, for values of T_{Rm}/T_S lower

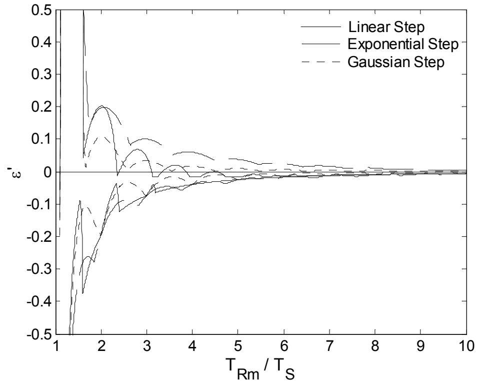

Fig. 4: Residual error ε', obtained after applying the bias removal correction (11)

than 2, the measurement results are dominated by the sampling period T_S. Conversely, for values of T_{Rm}/T_S greater than 10, the estimation error introduced by sampling and linear interpolation is negligible. Finally, it should be noticed that (4) is useful only when αT_S is not negligible with respect to the oscilloscope rise time, because the contribution of the analog circuitry to measurement errors cannot be easily corrected [1].

IV. ERROR ESTIMATION AND CORRECTION FOR FILTERED STEP SIGNALS

In order to analyze a more realistic situation, three new stimuli have been considered, obtained by feeding with a linear step, an exponential step, and a Gaussian step respectively the following second-order system:

$$H(\omega) = \frac{1}{1 + 2jD\dfrac{\omega}{\omega_0} + \left(\dfrac{j\omega}{\omega_0}\right)^2}, \tag{5}$$

with damping factor $D = 1/\sqrt{2}$, ω=2πf, where f is the frequency expressed in Hz, ω_0=2πf_0, and f_0=1/(2πT_R). Such a system introduces a moderate distortion on the considered waveforms, and it is characterized by a rise time of about 2.1T_R [1].

The analysis has shown that the second-order system has a regularizing effect, generating quite similar output signals even if the input step signals are appreciably different. By following the approach described in the previous section, the relative error has been estimated. First, ε has been evaluated and reported in Fig. 5, showing that the effect of sampling is quite similar for all of the considered filtered waveforms. Then, the mean value of ε has been fitted to (3) using the least squares method. For all of the considered waveforms a value of α close to 1.0 has been obtained. Thus (4) becomes:

$$T'_{Rm} = \sqrt{T_{Rm}^2 - T_S^2}, \quad 1 \le \frac{T_{Rm}}{T_S} \le 10, \tag{6}$$

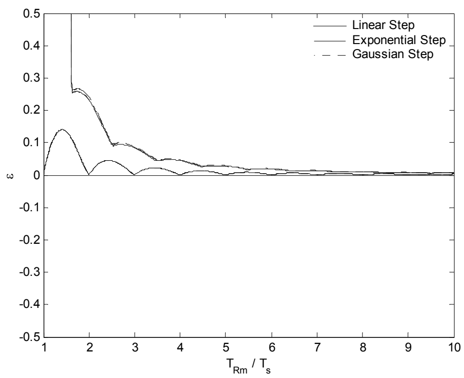

Fig. 5: Relative error introduced by sampling and interpolation on rise time measurements, obtained for the linear, exponential and Gaussian steps filtered by a second-order system.

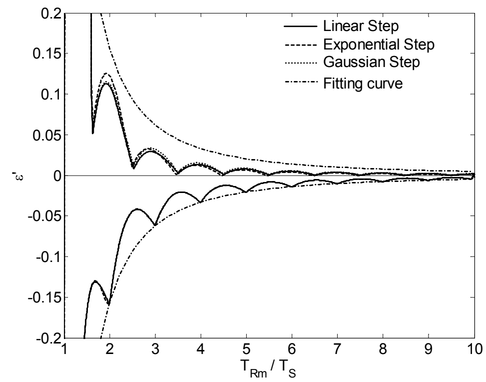

Fig. 6: Residual error ε', obtained after applying the bias removal correction (11) to the measured rise time, obtained for the linear, exponential and Gaussian steps filtered by a second-order system, and limiting curve.

whose application leads to the residual error curves reported in Fig. 6. As expected, the results are quite similar for all of the considered stimuli, and suggest that, the correction factor α=1.0 may be applied to most input signals. Finally, by applying fitting techniques to the maximum absolute local values of the residual error curves, the following hyperbolic law

$$\max\left(\frac{|T'_{Rm}-T_R|}{T'_{Rm}}\right)\leq\frac{0.5}{\left(T_{Rm}/T_S-0.3\right)^2},\qquad 1\leq\frac{T_{Rm}}{T_S}\leq 10. \tag{7}$$

has been derived, which provides an upper bound for the residual error magnitude and is also reported in Fig. 6.

V. CONCLUSIONS

The error introduced by sampling and linear interpolation on the rise time measurements of a step signal has been analyzed. In particular, a simple formula has been proposed which removes the error bias, and its usefulness and validity have been discussed. It has been shown that, differently from the analog case, a correction may be applied with advantage for a large class of input signals.

APPENDIX: EFFECT OF SAMPLING ON RISE TIME MEASUREMENTS

Let us call the estimates of $t_{10\%}$ and $t_{90\%}$ made by the DSO as $\hat{t}_{10\%}$ and $\hat{t}_{90\%}$ respectively. The effect of sampling and interpolation on the result of a rise time measurement can be theoretically evaluated by expressing $\hat{t}_{10\%}$ and $\hat{t}_{90\%}$ as a function of T_S, of the delay between the signal and the sampling instants, of the signal shape, and of the interpolation algorithm. In particular, in the case of linear interpolation, $\hat{t}_{10\%}$ is given by

$$\hat{t}_{10\%} = t_{10\%} + T_s \eta_{10\%} - T_s \left(\frac{f(t_n) - f(t_{10\%})}{f(t_n) - f(t_{n-1})} \right), \tag{A.1}$$

where $f(\cdot)$ is the mathematical law which describes the step signal to be sampled, t_{n-1} and t_n are the two consecutive sampling times such that $f(t_{n-1})<f(t_{10\%})<f(t_n)$, and $\eta_{10\%}=(t_n- t_{10\%})/T_s$ may be modeled as a random variable, uniformly distributed in [0,1] which keeps into account the lack of synchronization between the signal and the sampling times. Similarly, $\hat{t}_{90\%}$ is given by

$$\hat{t}_{90\%} = t_{90\%} + T_s \eta_{90\%} - T_s \left(\frac{f(t_m) - f(t_{90\%})}{f(t_m) - f(t_{m-1})} \right), \tag{A.2}$$

where t_{m-1} and t_m are such that $f(t_{m-1})<f(t_{90\%})<f(t_m)$, and $\eta_{90\%}=(t_m-t_{90\%})/T_s$. It should be noticed that $\eta_{10\%}$ and $\eta_{90\%}$ are not uncorrelated. In fact, by expressing the signal rise time T_R as follows

$$T_R = (k + \delta) T_S, \tag{A.3}$$

where k is an integer and $\delta \in [0,1[$, it can be shown that

$$\eta_{90\%} = \begin{cases} \eta_{10\%} + 1 - \delta, & 0 \le \eta_{10\%} < \delta \\ \eta_{10\%} - \delta, & \delta \le \eta_{10\%} < 1 \end{cases} \tag{A.4}$$

Thus, by using (A.1), (A.2) and (A.4), the DSO measured rise time $T_{Rm}=\hat{t}_{90\%} - \hat{t}_{10\%}$ may be expressed as

$$\begin{gathered} T_{Rm} = \begin{cases} (k+1-A)T_S, & 0 \le \eta_{10\%} < \delta \\ (k-A)T_s, & \delta \le \eta_{10\%} < 1 \end{cases} \\ A = \left(\frac{f(t_m) - f(t_{90\%})}{f(t_m) - f(t_{m-1})} - \frac{f(t_n) - f(t_{10\%})}{f(t_n) - f(t_{n-1})} \right) \end{gathered}, \tag{A.5}$$

where A expresses the effect of the signal shape on the rise time measurement when linear interpolation is used. The measurement error, normalized to T_{Rm} can then be expressed as

$$\varepsilon = \frac{T_{Rm} - T_R}{T_{Rm}} = \begin{cases} 1 - \dfrac{k+\delta}{k+1-A}, & 0 \le \eta_{10\%} < \delta \\ 1 - \dfrac{k+\delta}{k-A}, & \delta \le \eta_{10\%} < 1 \end{cases} \tag{A.6}$$

In order to validate the model accuracy, the mean value of ε has been also evaluated by numerically integrating (A.6). In particular, the relative deviation between theoretical results and simulations is lower than $9 \cdot 10^{-5}$ for $1 \le T_R/T_S \le 10$.

Finally, as the error on the estimation of both $t_{10\%}$ and $t_{90\%}$ is upper bounded by T_S, it results that $|T_{Rm}-T_R|<2T_S$. Consequently, the magnitude of the relative error ε, expressed as a function of T_{Rm}/T_S, is upper bounded by a hyperbolic curve, according to the following

$$|\varepsilon| \le \frac{2}{T_{Rm}/T_S} \tag{A.7}$$

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