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MEASUREMENT UNCERTAINTY AND METROLOGICAL CONFIRMATION IN QUALITY-ORIENTED ORGANIZATIONS

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Measurement Uncertainty and Metrological Confirmation in Quality–Oriented Organizations

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Abstract

The effect of measurement uncertainty on estimates and decisions performed under a regime of quality control and improvement, is considered in this paper. Standard statistical quality tools are analyzed such as control charts and instrument calibration procedures. Their performance is characterized under the assumption of both normally and uniformly distributed measurement uncertainty. Exact and approximate expressions are derived that allow the design of suitable procedures including the contribution of measurement uncertainty.

Index Terms

Test uncertainty ratio, metrological confirmation, statistical quality control, calibration.

I. INTRODUCTION

The extensive application of management models aimed at the assurance and improvement of company process quality levels has produced stimulating debates on the role of test and measurement in the development of a documented quality system. A prominent position has been taken by the widely accepted models described in the ISO 9000:2000 series of standards, which have been applied worldwide with steadily growing rates. Even in many other guidance norms and in topic-specific standards [1] the importance of test and measurement activities has always been highlighted. Moreover, the recognition by the international agencies, of the impelling need to regulate such a complicated matter has led to the publication of an European Pre–standard [2], known as the "Guide to the Expression of Uncertainty in Measurement," and of an important norm regarding the accreditation of laboratories [3], which now supersedes the former ISO/IEC Guide 25:1990.

According to the cited documents and to the best practices in managing instrumentation and product quality in a mature industrial environment, several tasks are commonly carried out, that require caution in dealing with uncertainty and related probabilistic risk assessment. This paper addresses the issues arising in a quality–oriented organization, from the use of measuring equipment involved in decision–making processes. At first, a brief description is made of the industrial practices requiring strict–sense measurements. Then, potential consequences of uncontrolled sources of uncertainty are analyzed both qualitatively and quantitatively. Finally, a numerical example is presented to provide further explanations about the applicability of the theoretical results described in this paper.

II. MEASUREMENTS AND QUALITY CONTROL

Running a program of statistical quality control and managing measurement equipment in accordance to the requirements of *metrological confirmation* [1], affects the procedures regarding process surveillance through

- control charts;
- conformance testing;
- calibration.

Many of the above cited techniques require prior evaluation of the principal statistical properties of the random variable $\mathbf{x}$ modeling the process outcome. Since uncertainty will affect the measured data, the manufacturing process behavior can be determined only up to a disturbing contribution related to the measurement procedure. If the random variable $\boldsymbol{\epsilon}$ represents the measurement uncertainty, then $\mathbf{y} \triangleq \mathbf{x} + \boldsymbol{\epsilon}$ is the random variable describing the measurement process outcome. Consequently, if $\boldsymbol{\epsilon}$ is zero–mean, under the common assumption of statistical independence, $\mu_y = \mu_x$ and $\sigma_y^2 = \sigma_x^2 + \sigma_\epsilon^2$, where μ and σ^2 represent the mean value and variance of the corresponding random variables, respectively. The sampling mean $\hat{\mu}_y \triangleq 1/N \sum_{n=0}^{N-1} \mathbf{y}[n]$, is commonly employed to estimate μ_y, where N is the number of processed samples and $\mathbf{y}[\cdot]$ is the sequence of measurement process outcomes. Similarly, the standard deviation of $\mathbf{y}$ is usually estimated using

$$\hat{\sigma}_y \triangleq \sqrt{\frac{1}{N-1} \sum_{n=0}^{N-1} (\mathbf{y}[n] - \hat{\mu}_y)^2}. \tag{1}$$

While the sampling mean is an unbiased estimator of the process mean, it is known that, when $\mathbf{y}$ is Gaussian, $(\hat{\sigma}_y/\sigma_y)\sqrt{N-1}$ is distributed as the square–root of a χ^2 random variable with $N-1$ degrees of freedom. Thus, it can be proved that [4]:

$$E\{\hat{\sigma}_y\} = \sigma_y c_4(N), \quad c_4(N) \triangleq \sqrt{\frac{2}{N-1}} \frac{\Gamma(N/2)}{\Gamma[(N-1)/2]}, \tag{2}$$

where $E\{\cdot\}$ is the expectation operator and $\Gamma(\cdot)$ is the so–called gamma function [5].

In the following subsections, it is shown how to include the effects of measurement uncertainty in quality–oriented practices aimed at process surveillance.

A. *Control Charts and Measurement Uncertainty*

Control charts are usually employed to monitor the average behavior and the variability of quality characteristics in a manufacturing or service process [4]. A widely applied pair of control charts is the $\overline{x}$–σ pair, which are used to control the behavior of sampled mean and sampled standard deviation of $\mathbf{x}$. Both control chart limits, that is the boundaries that if trespassed will trigger an *out-of-control* event, are frequently set by estimating the mean value and standard deviation of the random variable $\mathbf{x}$.

In fact, in a chart designed to monitor the process average ($\overline{x}$–chart), it is common to set the control limits around a given process mean value, μ_0 at a distance of $\pm 3\sigma_x$[1]. This choice sets also the probability of Type I errors. However, since σ_x is known only through (1), the contribution of $\boldsymbol{\epsilon}$ will affect the chart property with respect to probability of Type II errors and out-of-control *average run length* (ARL_1). The former quantity represents the probability of concluding that the process average is in control while it is out-of-control, while ARL_1 defines the average number of tests to be carried out before an out-of-control status is detected. Clearly, both the probability of Type II errors and ARL_1 are required to be low, so to improve the effectiveness of the control chart.

In order to appreciate the magnitude of the deviation from nominal behavior resulting from the effect of measurement uncertainty, assume the random variables to be Gaussian. Then, the probability of Type II errors, $\beta_{\overline{x}}(\cdot)$, that is the operating-characteristic curve, is given by:

$$\begin{aligned}\beta_{\overline{x}}(R) &\triangleq \mathrm{Pr}\{\text{Type II err.}\} = \mathrm{Pr}\{\mathrm{LCL} < \mathbf{y} \leq \mathrm{UCL} | \mu_x \neq \mu_0\} \\ &\simeq \Phi\left(3 - \frac{\delta}{\sigma_x\sqrt{1+\frac{1}{R^2}}}\right) - \Phi\left(-3 - \frac{\delta}{\sigma_x\sqrt{1+\frac{1}{R^2}}}\right)\end{aligned} \tag{3}$$

where LCL and UCL are the chart lower and upper control limits and $\Phi(\cdot)$ is the distribution function of a zero–mean unity–variance Gaussian random variable. Moreover, in (3), $\delta \triangleq \mu_x - \mu_0$, that is the deviation from the mean value that justifies the out-of-control status, and $R \triangleq \frac{\sigma_x}{\sigma_\epsilon}$ represents the *test uncertainty ratio* (TUR). Expression (3) holds approximately, since LCL and UCL are usually estimated in a prior chart design phase using the sampling mean and (1) applied to a large number of samples.

[1] A single observation is assumed in this paragraph. In the case of N independent observations, results still hold provided σ_x is replaced by $\sigma_x/\sqrt{N}$.

With the aim of determining how measurement uncertainty affects the probability of Type II errors, (3) has been plotted in Fig. 1 as a function of δ/σ_x assuming $R \in \{1, 1.5, 2, 5, \infty\}$, along with the corresponding values of $\mathrm{ARL}_1(R) = 1/[1 - \beta_{\overline{x}}(R)]$ reported on the right axis. The symbol '∞' applies when $\sigma_\epsilon = 0$, that is when measurement uncertainty can be neglected. The graph in Fig. 1 shows that for given δ and σ_x, by decreasing R, the effect of a given deviation is increasingly obscured by the presence of measurement uncertainty. In order to highlight the role of TUR in determining the behavior of (3) as a function of $\delta/\sigma_x < 0.5$, $\beta_{\overline{x}}(\cdot)$ has been expanded asymptotically assuming $R \rightarrow \infty$. Accordingly, the following results

$$\beta_{\overline{x}}(R) \simeq \beta_{\overline{x}}(\infty) + \frac{\delta}{2\sigma_x R^2}\left[\phi\left(3 - \frac{\delta}{\sigma_x}\right) - \phi\left(-3 - \frac{\delta}{\sigma_x}\right)\right], \qquad R \rightarrow \infty \tag{4}$$

in which $\phi(\cdot)$ is the probability density function of a zero–mean unity–variance Gaussian random variable and $\beta_{\overline{x}}(\infty) \triangleq \lim_{R\rightarrow\infty} \beta_{\overline{x}}(R)$, that is the probability of Type II errors when measurement uncertainty can be neglected. Simulation results show that the absolute difference between (4) and (3) is bounded by $6 \cdot 10^{-3}$ as long as $R > 3$ and $\delta/\sigma_x < 0.5$. Thus, under these assumptions, the deviation $\beta_{\overline{x}}(\infty) - \beta(R)$ vanishes approximately as $1/R^2$. Moreover, numerical simulations show that, for a given value of $R > 1$, (3) well approximates the probability of Type II errors also when uniformly distributed uncertainty is assumed.

A σ control–chart is designed in order to monitor the equivalence of the measured process standard deviation to a preset value σ_0, representing the standard deviation of the process when in statistical control, and usually determined in a prior phase of the chart design. Because of measurement uncertainty, such prior evaluation provides $\sigma_{y0} = \sigma_0\sqrt{1 + 1/R^2}$. Thus, since (1) is employed to evaluate estimate to be positioned on the chart, the resulting control chart will exhibit a central line equal to $\sigma_{y0} c_4(N)$. Moreover, by assuming LCL$= 0$, for a given value α of the probability of Type I errors, UCL=$\hat{\sigma}_y \chi_{N-1,1-\alpha}/\sqrt{N-1}$ results, where $\chi_{N-1,1-\alpha}$, is the $(1-\alpha)$–quantile of the square–root of a chi–square random variable with $N - 1$ degrees–of–freedom [4].

Consequently, the probability of Type II errors is given by (App. A):

$$\beta_\sigma(R) \triangleq X^2_{N-1}\left(\frac{1+\frac{1}{R^2}}{\lambda^2+\frac{1}{R^2}}\chi^2_{N-1,1-\alpha}\right) \tag{5}$$

where $X^2_{N-1}(\cdot)$ and $\chi^2_{N-1,1-\alpha}$ represent the probability distribution function and the $(1-\alpha)$–quantile of a chi–square random variable with $N-1$ degrees–of–freedom, respectively. Moreover, in (5), $\lambda \triangleq \sigma_x/\sigma_0$ where $\sigma_x \neq \sigma_0$, is the process standard deviation justifying the out–of–control condition. Observe that, when $R \to \infty$ and $\lambda \to 1$, (5) is equal to $1-\alpha$, as expected. In Fig. 2, $\beta_\sigma(\cdot)$ has been plotted assuming $N = 5$ and 25 for various values of the TUR. The curve labeled with $R = \infty$ corresponds to the absence of measurement uncertainty.

By following the same reasoning leading to (4) we obtain:

$$\beta_\sigma(R) \simeq \beta_\sigma(\infty) + \frac{\lambda^2-1}{R^2\lambda^4}\chi^2_{N-1,1-\alpha}x^2_{N-1}\left(\frac{\chi^2_{N-1,1-\alpha}}{\lambda^2}\right) \qquad R \to \infty \tag{6}$$

where $\beta_\sigma(\infty) \triangleq \lim_{R\to\infty}\beta_\sigma(R)$, and $x^2_{N-1}(\cdot)$ represents the probability density function of a chi–square random variable with $N-1$ degrees–of–freedom. Numerical investigations show that the absolute error between (5) and (6) is bounded by 10^{-2} as long as $R > 3$, $5\cdot10^{-3} < \alpha < 5\cdot10^{-2}$ and $5 \leq n \leq 100$. Moreover, as evidenced by (6), the deviation from $\beta_\sigma(\infty)$ vanishes as $1/R^2$.

B. Conformance Testing and Measurement Uncertainty

Conformance testing is the procedure by which a quality characteristic is measured against pre–set limits. These specifications may be a customer requirement or a legal obligation or part of the production regime. A common situation occurs when the product fails the test if the measured quantity is found to be external from a given interval. The contribution of measurement uncertainty may alter the final decision. In fact, because of the measurement procedure, an out–of–limit product may be wrongly accepted or a valid product may be wrongly rejected. Both situations can be characterized through corresponding probabilities of occurrence which define the so-called *consumer–risk* (CR) and *producer–risk* (PR) respectively defined as,

$$\text{CR} \triangleq \Pr\{\mathbf{x} \notin \mathcal{A} | \mathbf{y} \in \mathcal{A}\} \tag{7}$$

and

$$\text{PR} \triangleq \Pr\{\mathbf{x} \in \mathcal{A} | \mathbf{y} \notin \mathcal{A}\} \tag{8}$$

with $\mathcal{A} \triangleq (\mu_x - S, \mu_x + S)$, where S is the test specification limit. Under the assumption of $\mathbf{x}$ being normally distributed, CR and PR depend on whether the uncertainty is normal or uniformly distributed. By assuming uniformly distributed uncertainty the following results:

$$\text{CR} = \begin{cases} \text{erf}\left(\frac{1}{R}\sqrt{\frac{3}{2}} - \frac{\overline{S}}{\sqrt{2}}\right)\left(\frac{\overline{S}R}{2\sqrt{3}} - \frac{1}{2}\right) + \text{erf}\left(\frac{1}{R}\sqrt{\frac{3}{2}} + \frac{\overline{S}}{\sqrt{2}}\right)\left(\frac{\overline{S}R}{2\sqrt{3}} + \frac{1}{2}\right) & \\ -\frac{R\overline{S}}{\sqrt{3}}\text{erf}\left(\frac{\overline{S}}{\sqrt{2}}\right) - 2\frac{R}{\sqrt{6\pi}}e^{-\frac{1}{2}(\frac{3}{R^2}+\overline{S}^2)}\sinh\left(\frac{\overline{S}\sqrt{3}}{R}\right) & \overline{S} \leq \frac{\sqrt{3}}{2R} \\ & \\ \left(\frac{\overline{S}R}{2\sqrt{3}} + \frac{1}{2}\right)\left[\text{erf}\left(\frac{1}{R}\sqrt{\frac{3}{2}} + \frac{\overline{S}}{\sqrt{2}}\right) - \text{erf}\left(\frac{\overline{S}}{\sqrt{2}}\right)\right] & \\ +\frac{R}{\sqrt{6\pi}}e^{-\frac{\overline{S}^2}{2}}\left[e^{-(\frac{\overline{S}\sqrt{3}}{R}+\frac{3}{2R^2})} - 1\right] & \overline{S} > \frac{\sqrt{3}}{2R} \end{cases} \tag{9}$$

$$\text{PR} = \begin{cases} -\frac{1}{\sqrt{3}}\text{erf}\left(\frac{\overline{S}}{\sqrt{2}}\right)(R\overline{S} - \sqrt{3}) & \overline{S} \leq \frac{\sqrt{3}}{2R} \\ & \\ \left[\text{erf}\left(\frac{\overline{S}}{\sqrt{2}}\right) + \text{erf}\left(\frac{1}{R}\sqrt{\frac{3}{2}} - \frac{\overline{S}}{\sqrt{2}}\right)\right]\left(\frac{1}{2} - \frac{\overline{S}R}{2\sqrt{3}}\right) & \\ +\frac{R}{\sqrt{6\pi}}e^{-\frac{\overline{S}^2}{2}}\left[e^{(\frac{\overline{S}\sqrt{3}}{R}-\frac{3}{2R^2})} - 1\right] & \overline{S} > \frac{\sqrt{3}}{2R} \end{cases} \tag{10}$$

Conversely, by assuming normally distributed uncertainty, CR and PR are in the form of double integrals, which can not be integrated analytically. Then, using numerical approximations, and assuming $1 \leq R \leq 4$, and $1 \leq \overline{S} \leq 10$, with $\overline{S} \triangleq S/\sigma_x$, the following applies (App. B):

$$\begin{aligned} \text{CR} \simeq \frac{5}{\sqrt{2K_0}}\text{erf}\left(\sqrt{2}\cdot R\overline{S}\right)\left[1 - \text{erf}\left(\frac{100\overline{S} + 385R}{20\sqrt{K_0}}\right)\right] \\ \cdot e^{-\frac{1}{2}\overline{S}^2 + \frac{1}{2K_0}\left(77R\overline{S} + 50\overline{S}^2 + 30R^2\right)} \end{aligned} \tag{11}$$

where $K_0 \triangleq 38R^2 - 3R + 55$, and erf$(\cdot)$ is the error function [5]. Moreover,

$$\text{PR} \simeq 5\sqrt{\frac{5}{2K_1}}e^{-\frac{R\left(756R\overline{S}^2 + 1100\overline{S}\sqrt{2} - 605R\right)}{8K_1}}$$

$$\cdot\left\{\mathrm{erf}\left[\frac{100\overline{S}+151R^2\overline{S}+55\sqrt{2}R}{10\sqrt{K_1}}\right]-\mathrm{erf}\left[\frac{11\sqrt{2}R-20\overline{S}}{2\sqrt{K_1}}\right]\right\} \tag{12}$$

where $K_1 \triangleq 250+189R^2$. Simulation results show that (11) and (12) are accurate to within 4% to the corresponding probabilities evaluated without using approximations. By integrating the expressions arising from (7) and (8) or by using (9) – (12) directly, the conformance test parameter can be set, by also taking into account the effect of measurement uncertainty.

In order to illustrate the behavior of CR and PR, (9) through (12) have been graphed in Fig. 3 as a function of the TUR, assuming various values for the normalized threshold levels. Notice that, for a given value of TUR, the CR is larger when uniformly distributed uncertainty is considered. Thus, from this point of view, instruments exhibiting Gaussian behavior are to be preferred. Accordingly, refer to [7] and [8] for a description of experimental results reporting about commercially available instruments characterized by normal and/or uniformly distributed uncertainty. Finally consider that *guardbanding*–based techniques may be adopted for dealing with risks induced by measurement uncertainty. For a discussion about these methods refer to [9] and [10].

C. Calibration and Equipment Uncertainty

The need of verifying if an instrument obeys the requirements of metrological confirmation demands that the instrument be periodically tested and eventually calibrated. Accordingly, a suitable source of known accuracy is employed as the stimulus signal, so that the tested instrument uncertainty can be estimated. If such uncertainty remains inside an interval assuring the instrument working status, no calibration is performed. On the contrary, calibration may be required if, for some particular input values, the instrument uncertainty is larger than acceptable. Again, the intrinsic uncertainty of the stimulus source, can be at the origin of the two events leading to the consumer and producer risks. Moreover, by defining the test uncertainty ratio as the ratio between variances characterizing the instrument uncertainty and the source intrinsic uncertainty, (9)–(12) still hold true.

Once the evaluation of such probabilities can be mastered, the verification of instrument conformance can be achieved by following one of the many suggestions provided in the literature on how to choose specification limits [9], [10]. In this case, particular caution must be paid when dealing with both the consumer and producer risks. While passing an out–of–conformance instrument may lead to even potentially harmful consequences, being excessively conservative may put too many costs on the producer's side. Indeed, besides economic considerations, unnecessary calibrations might induce 'peaking and tweaking' effects in analog instruments [11]. The suggested technique of widening/shortening the recalibration interval according to the instrument previous conformance status, in accordance with the *simple response method* described in [1], is certainly affected by unwise choices regarding the consumer and producer risks.

III. A CASE STUDY IN MICROELECTRONIC MANUFACTURING

Wafer thickness is one of the most important quality parameters in the fabrication of Integrated Circuits (ICs) [12]. Although many new technologies, such as IC for smart cards, and packaging advances demand chips with a thickness down to 100 μm, 180 μm has been for a long time the standard substrate thickness usually required by microelectronic component manufacturers [13]. Generally, wafers are processed in small groups called lots. In Table I, 150 thickness values (expressed in μm) belonging to $M = 25$ different lots are reported. For each lot $N = 6$ distinct wafers are subjected to a quality control. The two rightmost columns in the table show the sampling mean and the sampling standard deviation, respectively, calculated using the values in the corresponding rows. Finally, at the bottom of the table, $\overline{\mu}_y = 180.4$ μm and $\overline{\sigma}_y = 6.5$ μm represent the average of the sampling means and of the sampling standard deviations, respectively.

In the example described hereafter, it is assumed that the measurement data reported in Table

I have been obtained using a non–contact instrument whose uncertainty is negligible compared with the standard deviation of the process. This assumption is absolutely reasonable because the standard uncertainty associated with the behavior of several instruments for measuring the wafer thickness is usually lower than 0.5 μm. In Fig. 4(a) the $\hat{\mu}$–chart of the process is plotted. The chart central line (CL) is set by the process specification whereas the Upper Control Limit (UCL) and the Lower Control Limit (LCL) are chosen equal to [4]:

$$\mathrm{LCL} = \mathrm{CL} - \frac{3}{\sqrt{N}}\frac{\overline{\sigma}_y}{c_4(N)} \quad \mathrm{UCL} = \mathrm{CL} + \frac{3}{\sqrt{N}}\frac{\overline{\sigma}_y}{c_4(N)} \tag{13}$$

where CL=180 μm, $N = 6$ and

$$\overline{\sigma} \triangleq \frac{1}{M}\sum_{m=1}^{M} \hat{\sigma}_y[m] = \frac{1}{M}\sum_{m=1}^{M} \sqrt{\hat{\sigma}_x^2[m] + \hat{\sigma}_\epsilon^2[m]} \tag{14}$$

so that $\overline{\sigma}_y/c_4(N)$ is an unbiased estimate of the standard deviation of the manufacturing process. In accordance to what stated in subsection 2.1, the probability of type II errors $\beta_{\overline{x}}(\cdot)$ depends on the amount of standard uncertainty affecting the measurements. If the standard uncertainty associated with the instrument is negligible (e.g. $\sigma_\epsilon = 0.5$ μm), it results that $\overline{\sigma}_y/c_4(N) = 6.9$ μm so that UCL $= 188.4$ μm and LCL $= 171.6$ μm. Notice that the test uncertainty ratio is large enough (e.g. $R = \sigma_x/\sigma_\epsilon \approx 14$) not to displace significantly the ideal chart limits drawn with continuous lines in Fig. 4(a) when $R = \infty$. However, if another instrument characterized by a standard uncertainty $\sigma_\epsilon = 3$ μm was chosen, the control chart limits would be wider ($\overline{\sigma}_y/c_4(N) = 7.5$ μm, UCL $= 189.2$ μm and LCL $= 170.8$ μm) due to the remarkable influence of measurement uncertainty ($R = 2.3$). This unavoidable enlargement of the control chart limits (dashed lines) causes an increase in the probability of Type II errors whose effect can be calculated directly by (3), simply replacing σ_x with $\sigma_x/\sqrt{N}$. For instance, if an out–of–control condition $\delta/\sigma_x = 2$ occurs, the probability of type II errors will pass from 3% to about 7%. Moreover, the risk of failing to detect an out–of–control condition becomes increasingly critical as the out–of–control deviation δ from the wished mean value (CL) grows. Similar considerations can be repeated about the $\hat{\sigma}$–chart shown in Fig. 4(b). In this case LCL=0 and UCL$=\overline{\sigma}_y \frac{\chi_{N-1,1-\alpha}}{\sqrt{N-1}}$, where $N-1=5$ and α has been set equal to 1%. As stated above, the continuous UCL line refers to the ideal case, whereas the dashed one is associated with the condition $R=2.3$. By applying (5), it results immediately that, for a given out–of–control standard deviation ratio λ, the probability of type II errors when $R=2.3$ is higher than the risk calculated when measurement uncertainty is negligible ($R=14$). For instance, if $\lambda=2$ the probability of type II errors $\beta_\sigma(\cdot)$ will pass from 42% to about 49%.

The data record reported in Table I can also be used to assess the influence of measurement uncertainty on both the consumer and the producer risks as explained in subsection 2.2. In fact, under the hypotheses that the measurement uncertainty is normally distributed and that the normalized test specification limit $\overline{S}=3$, both CR and PR can be estimated by (11) and (12) respectively. Similarly to the control chart case, both CR and PR become significantly higher when the TUR decreases. For instance, when $R=14$ the influence of measurement uncertainty is almost negligible (CR$=0.02\%$ and PR$=0.03\%$). Conversely, when $R=2.3$ it results that CR$=0.08\%$ and PR$=0.49\%$. Observe that the producer risk in the latter case is in excess of 16 times compared with the former one.

IV. CONCLUSIONS

In this paper, the effects of measurement uncertainties on quality–oriented measurements are analyzed. Directions are given on how to choose the accuracy of measurement instruments in order to reduce their influence when dealing with control charts, with conformance testing and with instrument calibrations carried out under programs of metrological confirmation in quality management systems. In addition, producer and consumer risks are recalled and new approximate expressions are presented under the hypothesis of normally and uniformly distributed uncertainties. The presented analysis proves that, in all cases, out–of–conformance or out–of–control risks related to measurement uncertainty can be reduced by setting suitably the test uncertainty ratio associated with the equipment employed.

V. APPENDIX A

DERIVATION OF EXPRESSION (5)

The probability of Type II errors, $\beta_\sigma(\cdot)$ is given by:

$$\begin{aligned}\beta_\sigma(R) &\triangleq \Pr\{\hat{\sigma}_y < UCL|\sigma_x \neq \sigma_0\}\\ &= \Pr\left\{\hat{\sigma}_y < \chi_{N-1,1-\alpha}\frac{\sigma_{y0}}{\sqrt{N-1}}|\sigma_x \neq \sigma_0\right\}.\end{aligned} \tag{A.1}$$

Thus,

$$\beta_\sigma(R) = \Pr\left\{\frac{\hat{\sigma}_y}{\sigma_y}\sqrt{N-1} < \chi_{N-1,1-\alpha}\frac{\sigma_{y0}}{\sigma_y}|\sigma_x \neq \sigma_0\right\}. \tag{A.2}$$

Since, in (A.2), the leftmost term in the brackets is distributed as the square–root of a chi–square random variable with $N-1$ degrees–of–freedom, we obtain (5), once observed that

$$\frac{\sigma_{y0}}{\sigma_y} = \sqrt{\frac{1+\frac{1}{R^2}}{\lambda^2+\frac{1}{R^2}}}. \tag{A.3}$$

VI. APPENDIX B

APPROXIMATIONS FOR CONSUMER AND PRODUCER RISKS

The consumer risk can be expressed as:

$$\begin{aligned}\text{CR} \triangleq &\int_{\mu_x-S}^{\mu_x+S}\int_{-\infty}^{\mu_x-S} f_{xy}(x,y)dxdy\\ &+\int_{\mu_x-S}^{\mu_x+S}\int_{\mu_x+S}^{\infty} f_{xy}(x,y)dxdy\end{aligned} \tag{B.1}$$

where $f_{xy}(\cdot,\cdot)$ is the joint probability density function of $\mathbf{x}$ and $\mathbf{y}$. By assuming independent zero–mean Gaussian random variables, from (B.1) we obtain

$$\text{CR} = \frac{1}{2\pi}\int_{\overline{S}}^{\infty} e^{-\frac{t^2}{2}}\gamma(R,\overline{S},t)dt \tag{B.2}$$

where $\gamma(R,\overline{S},t) \triangleq \left\{\text{erf}\left(R\frac{t+\overline{S}}{\sqrt{2}}\right) - \text{erf}\left(R\frac{t-\overline{S}}{\sqrt{2}}\right)\right\}$. Since the error function does not admit a primitive, $\gamma(\cdot,\cdot,\cdot)$ has been approximated using the expression

$$\gamma(R,\overline{S},t) \simeq \text{erf}\left(\sqrt{2}R\overline{S}\right) e^{-P_1(R)(t-\overline{S})} e^{-P_2(R)(t-\overline{S})^2}, \tag{B.3}$$

where $P_1(\cdot)$ and $P_2(\cdot)$ are two polynomials in R and the term erf$(\cdot)$ is justified by the need of forcing the approximating expression to be equal to $\gamma(R,\overline{S},t)$ for $t=\overline{S}$ and $t\to\infty$. By a least–squared-based numerical approach $P_1(\cdot)$ and $P_2(\cdot)$ have been identified as follows:

$$P_1(R) = 0.77R, \qquad P_2(R) = 0.38R^2 - 0.03R + 0.05. \tag{B.4}$$

Then, by inserting (B.4) in (B.3) and the resulting expression in (B.2) and by carrying out the integration, (11) results. A similar approach has been followed to derive (12).

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VII. AUTHORS' CURRICULUM VITAE

Paolo Carbone received the Ph. D. degree from the University of Padova, Padova, Italy in 1994. He joined the Electronics Engineering Department of the University Roma Tre in Rome, Italy, in 1994 as a Researcher. From 2000 to 2002 he was Associate Professor at the "Dipartimento di Ingegneria Elettronica e dell'Informazione," University of Perugia in Italy. Since 2003, he is full professor at the same Department. His major research interests are in the application of statistical signal processing for the improvement of electronic instrumentation performance and reliability and in the development of models for the analysis of signal quantization effects. Paolo Carbone currently serves as Associate Editor of IEEE Transactions on Circuits and Systems II.

David Macii received the M.S. degree in Electronic Engineering from the University of Perugia, Perugia, Italy in April 2000. In 2000 he spent a research period at the German Aerospace Institute (DLR) in Munich, Germany. Since January 2001 he has been pursuing the Ph. D. degree in Information Engineering at the "Dipartimento di Ingegneria Elettronica e dell'Informazione" of the University of Perugia. In 2002 he joined the Applied DSP and VLSI Research Group at the University of Westminster, London, where he spent a six-months research period. His research interests cover the design, implementation and characterization of data acquisition systems as well as the optimal management of measurement instruments in quality-oriented organizations.

Dario Petri received the Ph. D. degree in Electronic Engineering from the University of Padova, Italy, in 1990. From 1990 to 1992 he was at the "Dipartimento di Ingegneria Elettronica ed Informatica" of the same university as a research fellow. In 1992 he joined the "Dipartimento di Ingegneria Elettronica e dell'Informazione" of the University of Perugia, Italy, as an Associate Professor. From 1999 to 2002 he worked as a Full Professor of Measurement and Electronic Instrumentation at the same Department and the Chairperson of undergraduate and graduate degree study programs in Information Engineering.

At present, he is a Full Professor of Electronic Measurements at the "Dipartimento di Informatica e Telecomunicazioni" at the University of Trento. Research activities of Dario Petri are in the general areas of measurement science and technology, with particular interest to: data acquisition system design and testing, digital electronic system design and characterization, application of digital signal processing and statistical parameter estimation methods to measurement problems. Dario Petri is author and co-author of almost 100 papers.

VIII. Figure Captions

Figure 1. Probability of Type II errors and ARL_1 associated with an $\overline{x}$–control chart as a function of the normalized deviation from the mean under the effects of measurement uncertainty.

Figure 2. Probability of Type II errors associated with a σ–control chart as a function of the ratio between out–of–control and in–control process standard deviations under the effects of measurement uncertainty.

Figure 3(a) and **3(b).** Consumer (a) and producer (b) risks as a function of TUR and $\overline{S}$, assuming normal– (solid) and uniformly–distributed (dashed) uncertainties.

Figure 4. A $\hat{\mu}$–chart (a) and a $\hat{\sigma}$–chart (b) related to a wafer fabrication process. Both control charts are based on the data reported in Table I. Two sets of control lines are shown for $\sigma_\epsilon = 0.5$ μm (continuous lines) and $\sigma_\epsilon = 3$ μm (dashed lines), respectively.

IX. TABLES

	Wafer Thickness (μm)							
Wafer lots	1	2	3	4	5	6	tv	td
1	173.7	173.1	184.8	181.2	174.4	179.0	177.7	4.7
2	185.7	184.8	172.5	182.8	168.3	176.5	178.4	7.1
3	180.8	173.2	186.3	188.5	173.7	167.9	178.4	8.1
4	197.1	177.8	165.1	190.1	184.1	177.1	181.9	11.2
5	182.9	184.8	182.0	172.8	183.9	175.7	180.4	4.9
6	170.9	180.1	174.9	181.4	177.1	185.0	178.2	5.0
7	182.7	187.4	174.6	184.1	180.4	182.4	181.9	4.3
8	183.5	170.6	181.1	178.2	183.2	186.2	180.5	5.5
9	176.4	183.4	177.6	197.5	181.4	182.0	183.0	7.6
10	181.6	168.6	186.8	186.0	181.8	179.0	180.6	6.6
11	175.8	169.9	179.2	174.0	194.6	179.4	178.8	8.5
12	180.1	182.1	187.1	185.7	164.1	182.0	180.2	8.3
13	182.9	179.0	176.7	184.9	182.4	188.2	182.4	4.1
14	188.3	172.1	180.5	185.3	182.0	185.6	182.3	5.7
15	185.4	178.0	182.8	168.0	184.6	170.5	178.2	7.5
16	161.5	175.9	187.8	190.8	175.9	180.8	178.8	10.4
17	182.0	173.7	184.3	168.7	186.2	178.4	178.8	6.7
18	185.8	181.7	178.0	187.8	181.2	184.0	183.1	3.5
19	179.9	169.6	170.4	172.2	185.9	177.9	176.0	6.4
20	186.0	182.2	175.2	182.7	186.7	187.9	183.4	4.6
21	185.4	165.8	182.3	186.8	189.3	178.7	181.4	8.5
22	189.1	183.7	173.0	185.7	179.5	169.7	180.1	7.5
23	188.6	182.4	182.0	180.3	189.2	189.8	185.4	4.3
24	186.7	185.3	187.7	173.5	181.6	183.1	183.0	5.2
25	168.4	175.2	181.7	179.2	170.0	184.0	176.4	6.3
							tf =180.4	te =6.5

Table I. Wafer thickness expressed in μm: 150 values related to 25 distinct 6–piece lots are listed.