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MEASUREMENT UNCERTAINTY IN DECISION ASSESSMENT FOR TELECOMMUNICATION SYSTEMS

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MEASUREMENT UNCERTAINTY IN DECISION ASSESSMENT FOR TELECOMMUNICATION SYSTEMS

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This paper deals with the effects of measurement uncertainty on decision making. In particular, conformance tests for communication systems equipment are considered, with respect to both consumer and producer risk and the effects of such parameters on the overall costs.

1. Introduction

The recent evolution of industrial scenarios, which tends to focus competition on efficiency and quality control, makes measurement a very important field, subject to both research and normative activities. This importance is due to the fact that measurement results are used to make choices and decisions, which deeply affects the efficiency and the profits of any industrial activity. A typical example is the conformance test, which is used to decide if a product should be put on the market or discarded.

Telecommunication industry is not an exception to this trend. In fact, while earlier standards report only the specification and requirements of a given system, the most recent ones are also focused on the measurement procedures [1]. Recently, more attention has been also given to measurement uncertainty, which is now considered an important part of measurement results. In particular, [2] describes how to evaluate the uncertainty for various kind of measurements performed on communication system equipment, reporting for each considered case the most important sources of uncertainty. Furthermore, various methods to reduce the measurement uncertainty can be found in the literature for various application fields [3].

However, to the knowledge of the authors, the effects of measurement uncertainty on a decision making process have not been deeply investigated yet, at least in the telecommunication field. This paper addresses such effects on the outcomes of a conformance test, with respect to both Producer Risk and Consumer Risk and the involved costs. Finally, the effects of measurement uncertainty on a conformance test performed on a digital receiver are discussed.

2. Classical approach to the conformance testing

A conformance testing is used to determine if a given product satisfies a set of requirements, which may be defined by the consumer, by the producer or by regulations and norms. For each of the relevant properties of the product a nominal value X is reported along with a tolerance T, which defines the maximum acceptable deviation of the considered property from the nominal value. The conformance testing is usually performed by measuring a given product and by verifying that the measurement result x_M does not differ excessively from the nominal value. In particular, the product is accepted when $x_M \in S_T=[X-T, X+T]$, and rejected when $x_M \in S_{NT}=(-\infty,X-T]\cup[X+T,\infty)$. While straightforward, this approach, does not keep into account the measurement uncertainty, and is acceptable only when the measurement uncertainty is negligible with respect to the tolerance T. In fact, the measurement uncertainty may alter the conformance testing results, leading to incorrect decisions. Let us define x as the result of an ideal measurement not affected by uncertainty sources, where x differs from X by a deviation due to the imperfections of the manufacturing process, and let us assume that $x_M=x+e$, where e is a deviation caused by the uncertainty sources introduced in a real measurement environment. Two cases are possible, where the measurement uncertainty leads the conformance testing to incorrect results. The first case happens when a non conforming product is accepted. The probability of such an event, known as Consumer Risk (CR), is defined as:

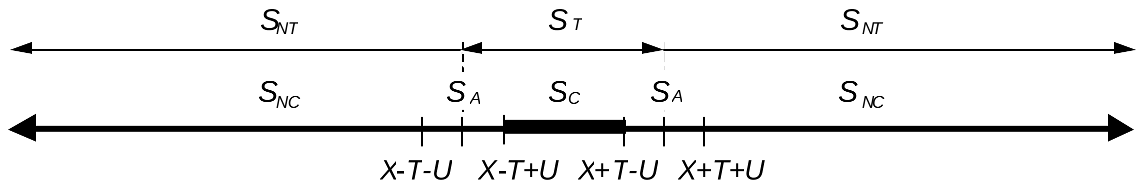

Fig. 1: decision sets adopted for the modern approach to the conformance test

$$CR=\Pr\{x\in S_{NT} \mid x_M \in S_T\}, \tag{1}$$

The second case happens when a conforming product is wrongly discarded due to the measurement uncertainty. The probability of such an event, known as Producer Risk (*PR*), is:

$$PR=\Pr\{x\in S_T \mid x_M \in S_{NT}\}. \tag{2}$$

It should be noticed that *PR* and *CR* correspond to two dual events, which are both undesirable for an industrial subject. In particular, the *PR* has a direct impact on the overall costs, because a conformant product is not sold. On the other hand, *CR* affects both the consumer and the producer, due to the loss of image and eventual costs sustained to replace a defective product. Both *PR* and the *CR* can be theoretically evaluated by making assumptions on the stochastic properties of the measurement uncertainty. Numerical expressions for *CR* and *PR* have been derived in [3], for uniformly and Gaussian distributed measurement uncertainties. In particular, when both x and e are Gaussian distributed and statistically independent, *CR* and *PR* depend on the process capability $T'=T/s_x$ and the testing uncertainty ratio $R=s_x/u_e$, where s_x is the standard deviation of x and u_e is the standard deviation of e. These results show that *CR* and *PR* can be varied by acting both on the process capability or on the measurement system.
The overall costs of a conformance test can be evaluated as the sum of the cost of the measurement system, which increase if u_e is lowered, and the cost of the products wrongly accepted or discarded, which depend on both T' and u_e. In particular, improving the process capability reduces the number of non conformant products, at the price of higher production costs. Conversely, both *PR* and *CR* can be reduced by improving the measuring system, that is by lowering u_e, but such a strategy may be ineffective if the process capability is too low. Thus, in order to effectively manage the costs of a conformance test, an optimal balance between measurement uncertainty and process capability is needed.

3. Modern approach to the conformance testing

According to the evolution of measurement scenarios [3], the uncertainty is being considered as an essential part of a measurement result and as a cost in itself, both for the increased cost of a low uncertainty measurement system and for the costs of erroneous decisions caused by the uncertainty sources.
When measurement uncertainty is not negligible, the measurement scenario is better represented by Fig.2, where two new zones appear, both centered around the ideal decision thresholds, defining the so called ambiguity set $S_A=[X-T-U,\ X-T+U)\cup[X+T-U,\ X+T+U)$ respectively, where U is the expanded measurement uncertainty [3][5]. The sets S_T and S_{NT} introduced in the previous section are now respectively replaced by the conformance set $S_C=[X-T+U,\ X+T-U]$ and the non-conformance set $S_{NC}=(-\infty,\ X-T-U]\cup[X+T+U,\ \infty)$.
When $x_M\in S_C$ the product is most likely to be conformant, that is $x\in S_T$. Under the same conditions, if $x_M\in S_{NC}$ then it is probable that $x\in S_{NT}$, and the product is not conformant. However, when the measurement result lies in S_A, the result of the conformance test is ambiguous, because in this region the measurement uncertainty is dominant with respect to the variation of the measurand caused by the production process. It should be noticed that when $U=T$, S_C reduces to an empty set, and the conformance test cannot provide reliable results.
An important consequence underlying this approach is that the costs associated to the measurement uncertainty should be afforded by which is attempting to prove the conformance or

the non-conformance of a given product. Hence, a producer performing a conformance test should accept only products belonging to the S_C set. Conversely, when a subject is attempting to prove that a product is out of limits, it should prove that it belongs to the S_{NC} set. Such an approach has been introduced in order to encourage the adoption of more effective measuring politics, thus reducing the probability of erroneous decisions and improving the overall quality and efficiency of the manufacturing processes. An important consequence is that both *CR* and *PR* are no more uniquely defined, but depend on the testing purposes. In fact, if a producer is attempting to prove the conformance of a product, the corresponding consumer risk and producer risk are defined as:

$$CR_C = \Pr\{ x \in S_{NT} \mid x_M \in S_C \}, \quad (3)$$

$$PR_C = \Pr\{ x \in S_T \mid x_M \notin S_C \}. \quad (4)$$

In this case, when U is increased CR_C tends to become smaller than CR, and PR_C tends to become greater than PR. If e can be modeled as an amplitude limited random variable, such as the uniformly distributed one, CR_C can be null. The producer should be moved to reduce the measurement uncertainty, in order to reduce PR_C. In particular, the target uncertainty U should be chosen as the value which minimizes the sum of the costs of the measurement system and the costs introduced by the producer and consumer risks.
Conversely, when a consumer or an organization is attempting to prove the non-conformance of a product, the corresponding risks CR_{NC} and PR_{NC} are given by:

$$CR_{NC} = \Pr\{ x \in S_{NT} \mid x_M \notin S_{NC} \}, \quad (5)$$

$$PR_{NC} = \Pr\{ x \in S_T \mid x_M \in S_C \}. \quad (6)$$

In this case, when U is increased PR_{NC} tends to become smaller than PR, and CR_{NC} tends to become greater than CR. If e can be modeled as an amplitude limited random variable, PR_{NC} can be equal to zero. The consumer should be moved to reduce the measurement uncertainty, in order to reduce CR_{NC}. In particular, the target uncertainty U should be chosen as the value which minimizes the sum of the costs of the measurement system and the costs introduced by the risk. It should be noticed that, also in this case, both the producer risk and the consumer risk depend also on the process capability. In particular, reducing the measurement uncertainty may not be particularly advantageous if the process capability is high, that is if only a very low percentage of products is out of limits.

4. Measurement uncertainty in a practical telecommunication scenario

An important parameter in measuring the performance of Digital Communication Systems is the Bit Error Rate (*BER*). A *BER* measurement system transmits a known sequence of bits to a receiver, and estimates the *BER* as the ratio of the number of bit incorrectly received and the total number N of transmitted bits. It can be shown that this estimator is unbiased, and for large values of N tends to be Gaussian distributed with a standard deviation u_{BER} given by [2]:

$$u_{BER} = \sqrt{BER(1-BER)/N} \quad (7)$$

BER is also used for indirect measurements. In particular, in the following the measurement of the sensitivity of a digital receiver will be considered. The test setup, summarized in Fig. 2, measures the sensitivity of a receiver as the minimum input RF signal power such that at the output of the receiver under test a threshold *BER* value b is measured. Such a measurement is affected by various uncertainty sources which can be grouped in two main contributions uncorrelated with each other. The first one, whose relative standard uncertainty we will call u_{RFSYS}, is related to the components of the measurement system placed before the receiver, like the signal generator level uncertainty or the mismatching introduced by the connectors [2]. The second contribution, whose relative standard uncertainty we will call u_{RFBER}, is introduced by the *BER* estimation [2], and can be obtained from u_{BER} as follows:

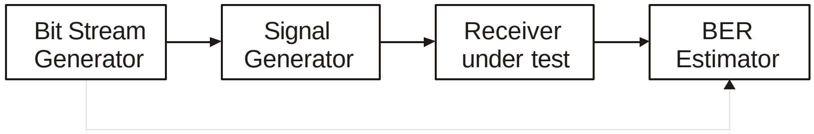

Fig. 2: Test Setup for measuring the sensitivity of a digital receiver

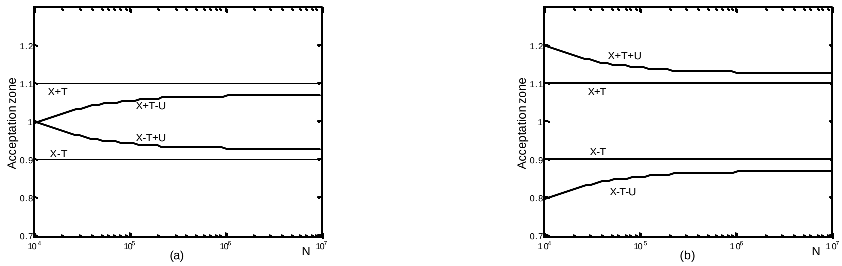

Fig. 3: Acceptation zones, for the conformance test (a) and for the non-conformance test (b) of the sensitivity of a digital coherent receiver.

$$u_{RFBER} = \frac{u_{BER}}{BER'(SNR_b^*) \cdot SNR_b^*}, \qquad (8)$$

where SNR_b^* is the Signal to Noise Ratio (SNR) at the input of the receiver which for the considered digital modulation technique corresponds to the *BER* value *b*, and $BER'(SNR_b^*)$ is the first derivative of the law which expresses the *BER* as a function of the receiver input *SNR*, evaluated in SNR_b^*. Finally, the relative uncertainty u_{RF} is obtained as follows [2][5]:

$$u_{RF} = \sqrt{u_{RFSYS}^2 + u_{RFBER}^2}\ . \qquad (9)$$

Such results may be used to estimate the amplitude of the intervals of acceptation to be used in a conformance testing. Figg. 3(a) and 3(b) report the positions of the corresponding thresholds, respectively for the conformance testing and for the non-conformance testing, as a function of *N*. Both figures have been obtained by assuming a coherent digital modulation like the one used in GSM systems, with $b=10^{-2}$, $X=1$, $T=10\%$, $u_{RF1}=1\%$, and a coverage factor $k=3$ [2][5]. It can be seen that the decision intervals approach the ideal ones as *N* is increased, but do not converge completely because of the constant contribution u_{RFSYS}.

5 Conclusions

The effects of measurement uncertainty on the results of conformance tests have been analyzed. In particular, the effects on the consumer risk and on producer risk have been discussed, both for the classical approach to the conformance testing and for the modern approach, which takes into account measurement uncertainty. Furthermore, a practical case has been considered, showing how for a sensitivity test performed on a digital receiver the acceptation intervals are affected by both the measurement system and the *BER* estimation uncertainties.

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