

The Condorcet paradox: an experimental approach to a voting process

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Abstract

This paper analyses the effects played by rules within a coordination game. The starting point is constituted by the wide field of Public Choice theories. More precisely the focus of the research is on the stability of the voting process. The experiment is build on a game played through computers and the experimental subjects must perform some choices that can led to different individual and collective solutions. The game that they play is based on a set of rules that must be voted by the players themselves before a new session of the experiment will be run. The idea is to verify the degree of stability of the collective choices (log-rolling phenomena).

FIRST DRAFT

1. Introduction

This paper is on the Condorcet paradox. More precisely is an experimental investigation on the stability of the voting process. The idea is to test the well known phenomenon of cyclical voting that should arise whenever the voters have double peaked preferences.

The Condorcet paradox is: “a situation described by the Marquis de Condorcet in the late 18th century, in which collective preferences can be cyclic (i.e. not transitive), even if the preferences of individual voters are not. This appears paradoxical, because it means that majority wishes can be in conflict with each other. This paradox can be explained away by the fact that in that case the majorities are made up of different groups of individuals. The paradox is highlighted by the Condorcet method of voting, which will fail to determine a winner in such a situation — an alternate technique must then be used”¹.

A simple way to describe the Condorcet paradox is to use a numerical example like the following (Rouncefield, Green, 2003):

Three players: A, B and C

$\Pr(\text{A outscores B}) = 5/9$

$\Pr(\text{B outscores C}) = 5/9$

$\Pr(\text{A outscores C}) = 4/9$

i.e. A is better than B i.e. B is better than C i.e. A is not better than C, which means that the results are not transitive.

An example of a real game that produces a Condorcet type situation is the so called Chinese Dice game (Green, 1981) The dice of this game are marked in this way:

Die A) 6, 6, 2, 2, 2, 2

Die B) 5, 5, 5, 5, 1, 1

Die C) 4, 4, 4, 3, 3, 3

For these:

$\Pr(\text{A outscores B}) = 5/9$

$\Pr(\text{B outscores C}) = 6/9$

$\Pr(\text{A outscores C}) = 3/9$

Also this game produces results which are not transitive and therefore are cyclical.

The most important application of the Condorcet paradox is the study of the voting systems. The paradox arises every time the voters have a preferences structure which is individually transitive but collectively intransitive. For example imagine three voters that

have the following preferences structures over three alternatives a_1, a_2, a_3 (e.g. three different quantities of some public good):

Voter	Preference
1	$a_1 \succ a_2 \succ a_3$
2	$a_2 \succ a_3 \succ a_1$
3	$a_3 \succ a_1 \succ a_2$

Assuming that the voters give their support to the first and second choice while never vote for the third choice, then a winner cannot exist for this profile if all the alternatives are individually voted.

In literature there are many examples of real situations where the Condorcet paradox takes place. I shall restrain myself to quote only Kurrilid-Klitgaard (2001) who describes the case of the existence of a real cyclical majority in a poll of Danish voters' preferred prime minister, using pair-wise comparison during the elections of the Danish prime minister in 1994. Kurrilid-Klitgaard (2001) is also useful for a short review of some articles based on empirical evidences of the Condorcet paradox.

Conversely looking to the experimental literature there are not many examples of experiments done on the voting process. The focus of the experimental researches is mainly concerned with the effects produced on the collective choices by different voting systems, instead than on the investigation of the role played by different preferences structures. Just as an example of an experimental study on the role played by different voting rules one could see Forsythe (1991) that analyses a three ways elections model.

The main attention is here concentrated on the central assumption made by the Condorcet paradox, i.e. on the effects produced by double peaked preferences on the stability of the voting process. This is a topic that is very difficult to investigate using a "traditional" empirical approach because the only practical way to check the preferences structure is to ask to the voters to declare spontaneously their wishes towards a given topic. Obviously the voters can have strategic reasons to make false declarations and in any way the real behaviours are not observable so there is no way to check the truthfulness of the individual declarations.

A way to overcome these limits is to use the experimental approach, which allows to simulate in an artificial environment a voting process. To simulate a voting process able to investigate on the effects produced by the Condorcet paradox requires three main ingredients:

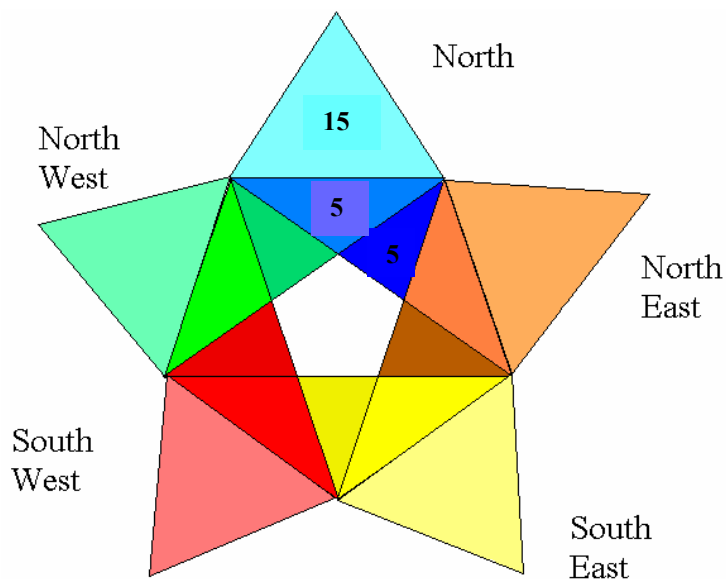
¹ http://en2.wikipedia.org/wiki/Condorcet's_paradox

a payoffs structure that links the utility of the experimental subjects to the voting choices, an artificial structure of preferences coherent with the payoffs-utility configuration and a social environment. The experiment here discussed owns all these characteristics.

2. The experimental design

The experiment is a repeated game played by five players. Each round is divided in two phases: the voting stage and the playing stage. In the voting stage the experimental subjects must choose (and vote) a rule that will be used in the game stage. To play the game the subjects must rotate one among four geometric figures that shape a pentagon star. In fig. 2.1 is reported the form used for the game.

Fig. 2.1 The pentagon star



The first geometrical figure is the large pentagon star which includes the five external triangles, the second figure is the inscribed pentagon which is made by the five intermediate triangles, finally the third figure is the small star made by the five inner triangles. Each figure rotates anti-clockwise, independently from the other ones, and with single steps. The single step rule means that to move one triangle from its initial position back to the starting setting

requires five moves (steps). For example to take the blue external triangle from the North position to the South West one needs three moves.

Each of the five players occupies a position indicated by a cardinal point. Subject 1 is in position North, subject 2 is in position North East, subject 3 is in position South East, subject 4 is in position South West and finally subject 5 is in position North West. To each player corresponds a colored geometrical surface which is the sum of three triangles: the blue area for subject 1, the orange area for subject 2, the yellow area for subject 3 (yellow), the red area for subject 4 and the green area for subject 5.

To the colored surfaces correspond the payoffs areas. The players gain a final prize which is the sum of the values reported on each triangle. Only three triangles have numbers on them, therefore to win something one must rotate the figure which takes one of the numbered triangles in her/his payoff area. For example if it is the turn of player North West the most rational move is to rotate the external star which will attribute to her/him a payoff of 15 points.

The general ingredients of the experiment are the following:

- five players, four groups of players;
- anonymity;
- a set of rounds – each round is divided into two phases: phase1 voting the rules; phase2 move the wheels accordingly with the rule voted by the majority; one rotation per player;
- the positions of the players are assigned randomly;
- after 3 voting sessions without a majority the experiment stops and the subjects win a fixed (small) reward.

The number of players can be obviously changed but a total of 25 subjects for each experimental session build up a reasonable sample. Similarly the anonymity condition is not a strict one and can be relaxed. In the experiments here discussed has been maintained to have a “cleaner” experimental outcome. When the subjects have no way to identify their partners the results are less affected by psychological uncontrolled factors like for example some form of sympathy or antagonism between two or more participants.

The rules that must be voted in the first phase of the experiment are the following:

- (A) - All the wheels can be rotated
- (I) - only the Internal wheel can be rotated
- (E) - only the External wheel can be rotated

The rules used for the experiment are intended to build a system of artificial preferences that models a Condorcet paradox situation. More precisely the artificial preferences needed are the following:

- Player North (N) $I \succ A \succ E$
- Players North and South East (NE; SE) $E \succ I \succ A$
- Players North West and South West (NW; SW) $A \succ E \succ I$

To obtain the desired artificial preference system the rules must be integrated with a special condition for player North. More precisely to be fair and to be coherent with the artificial preferences system the payoffs scheme must take this structure:

- each point is converted in Euro Cents
- 1 cent for player North
- 9 cents for players North East and South East
- 3 cents for Players North West and South West
- special payoff for player North: if the rule is A s/he will win 22 points if the light blue triangle goes to occupy the position South West.

The artificial preference structure just described holds only for the first move, i.e. in a one-shot game setting. To explore the dynamical solution of the game – assuming that each player moves only once for a total of 5 moves per round – is useful to write the numerical solution. Tab. 2.1 reports the solution of the game (assuming rational players). Looking to tab. 2.1 is immediately clear that from a social point of view the most efficient rule is the “all wheels can rotate”. At the same time it is also evident that the all wheels rule is the worst for both the North East and the South East players that never win when this rule is at work. The obvious consequence is that the East players should try to contrast the all wheel rule.

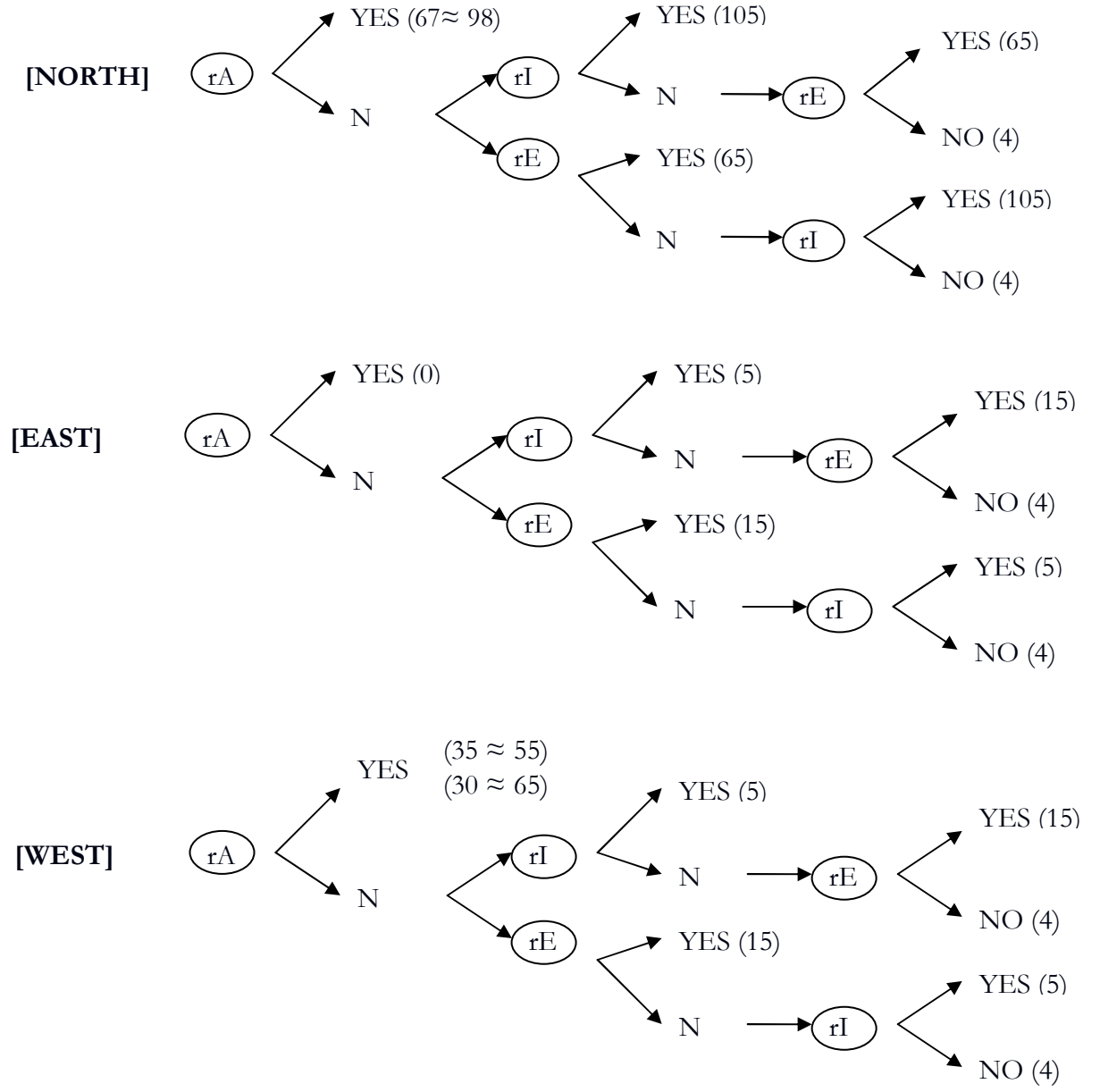
An even clearer demonstration of the dominance of the all wheels rule is given by the dynamic solution of the game. The solution of a session made of five moves (one for each player) is reported in fig. 2.2. It is important to underline that at the beginning of the experiment the players do not know the voting agenda, i.e. they do not know ex ante if they should vote first for rule all, then for rule external and then for the rule internal or for some different sequence of the rules. In particular they do not know what rule will follow the first voting session.

Tav. 2.1 Rational moves													
	Moves					Payoff							Pay Tot
Player	NE	SE	SO	NO	N	N	NO	SO	NE	SE			per round
Rule All	E	E	M	I	M	98	35	65	0	0			198
	E	E	M	I	I	98	35	65	0	0			198
	E	E	I	M	M	98	35	65	0	0			198
	E	E	I	M	I	98	35	65	0	0			198
	E	M	E	I	M	81	55	50	0	0			186
	E	M	E	I	I	81	55	50	0	0			186
	E	I	E	M	M	81	55	50	0	0			186
	E	I	E	M	I	81	55	50	0	0			186
	M	E	E	I	M	91	45	50	0	0			186
	M	E	E	I	I	91	45	50	0	0			186
	M	M	E	I	E	67	45	35	0	0			147
	M	M	I	E	E	77	35	35	0	0			147
	M	I	M	E	E	72	45	30	0	0			147
	M	I	I	E	E	72	45	30	0	0			147
	I	E	E	M	M	91	45	50	0	0			186
	I	E	E	M	I	91	45	50	0	0			186
	I	M	M	E	E	72	45	30	0	0			147
	I	M	I	E	E	72	45	30	0	0			147
	I	I	E	M	E	67	45	35	0	0			147
	I	I	M	E	E	77	35	35	0	0			147
Average													172,8
Rule External	E	E	E	E	E	65	15	15	15	15			125
Rule Internal	I	I	I	I	I	105	5	5	5	5			125

The final situation that emerges from the experimental design is not the one described by the Condorcet paradox because the cyclical nature of the voting process holds only in a one shot perspective while in a strategic perspective the result should be of a steady state with the dominant rule always voted by the majority.

The experiment models a situation where the static preferences are double peaked and should take to a cyclical process of voting but the dominant dynamic strategy is stable so we expect that a rational output for the game is to lock in the “all wheels” rule.

Tab. 2.2 The dynamic solution



Sequences:

North $NO_A YES_I YES_E$

North 65

East $NO_A NO_I YES_E$

East 15

West $NO_A NO_I YES_E$

West 15

North $NO_A NO_E$

North 65

East $NO_A YES_E$

East 15

West $NO_A YES_E$

West 15

North	YES _A	$67 \approx 98$
East	NO _A	East 0
West	YES _A	West $35 \approx 55$; $30 \approx 65$

3. The results

This section of the paper has not yet fully developed so only some data are reported.

Tab. 3.1 Results from the first experiment

Group 1

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T
N	0	1	0	0	1	1	0	1	1	0	1	1	0
NE	0	0	0	1	0	0	0	0	1	1	0	1	0
SE	1	0	0	1	0	0	1	0	0	1	0	0	1
SO	1	1	1	1	0	1	1	0	1	1	1	1	1
NO	1	1	1	1	0	1	1	0	0	1	0	0	1
accepted	T	I	-	T	-	E	T	-	E	T	-	E	T

Group 2

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T
N	1	1	1	1	1	1	1	1	1	1	1	1	1
NE	1	0	0	0	0	1	0	0	1	0	0	1	0
SE	1	0	1	0	0	1	0	0	1	0	0	1	0
SO	0	1	1	1	0	0	1	0	0	1	0	0	1
NO	1	0	1	1	0	0	1	0	0	1	0	0	1
accepted	T	-	E	T	-	E	T	-	E	T	-	E	T

Group 3

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E
N	1	1	1	1	0	1	1	0	1	1	0	0	1	0	1
NE	0	1	0	0	1	1	0	0	1	0	0	1	0	0	1
SE	1	0	1	1	0	1	1	0	1	0	0	1	1	0	1
SO	1	0	0	1	0	0	1	1	0	1	1	0	1	0	0
NO	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0
accepted	T	-	E	T	-	E	T	-	E	T	-	-	T	-	E

Group 4

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E
N	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1
NE	1	1	1	0	0	1	0	0	1	0	0	1	0	0	1
SE	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1
SO	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0
NO	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
accepted	T	-	E	-	-	E	T	-	E	T	-	E	T	-	E

Tab. 3.2 Results from the second experiment

Group 1

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E
N	1	0	0	1	1	0	1	1	0	1	1	0	1	1	0	0	1	0	0	1	0
NE	1	0	1	1	1	1	0	0	0	1	1	1	0	1	0	0	1	1	0	0	1
SE	0	0	0	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1
SO	1	0	1	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	0	0
NO	0	1	1	0	1	1	1	0	0	1	0	0	1	1	1	1	0	1	1	0	1
accepted	T	-	E	T	I	E	T	-	-	T	I	E	T	I	E	-	I	E	-	-	E

Group 2

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	
N	1	1	0	0	1	1	1	1	0	1	1	0	0	1	0	0	1	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1
NE	1	1	1	1	0	1	1	0	1	0	1	1	0	1	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	
SE	1	0	0	1	0	0	1	0	1	1	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	
SO	0	1	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	
NO	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	1	1	0	0	1	0	0	1	
accepted	T	I	-	T	-	-	T	-	-	T	-	-	-	-	-	T	-	-	T	-	-	-	-	E	T	-	-	T	-	-	T	

Group 3

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	
N	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1	0	1	1	0	1
NE	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	
SE	1	0	0	1	0	1	1	0	0	1	0	0	1	0	0	1	0	1	1	0	1	0	0	1	0	0	1	
SO	1	0	1	1	0	1	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	0	0	1	1	0	
NO	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1	0	1	0	0	1	0	0	
accepted	T	-	-	T	-	E	T	-	-	T	-	-	T	-	-	T	-	E	T	-	-	T	I	-	T	I	E	

Group 4

VOTE FOR	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	T	I	E	
N	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1
NE	1	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	
SE	1	1	0	0	1	1	0	1	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	1	0	1	
SO	1	0	0	1	0	0	1	0	0	1	1	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	
NO	1	0	1	1	1	0	1	0	0	1	0	0	1	0	1	1	0	0	1	0	0	1	0	0	1	0	0	
accepted	T	-	E	-	-	E	T	-	E	-	-	E	-	-	E	T	-	-	T	-	-	T	-	-	-	-	T	

Fig. 3.1 Vote series in exp2

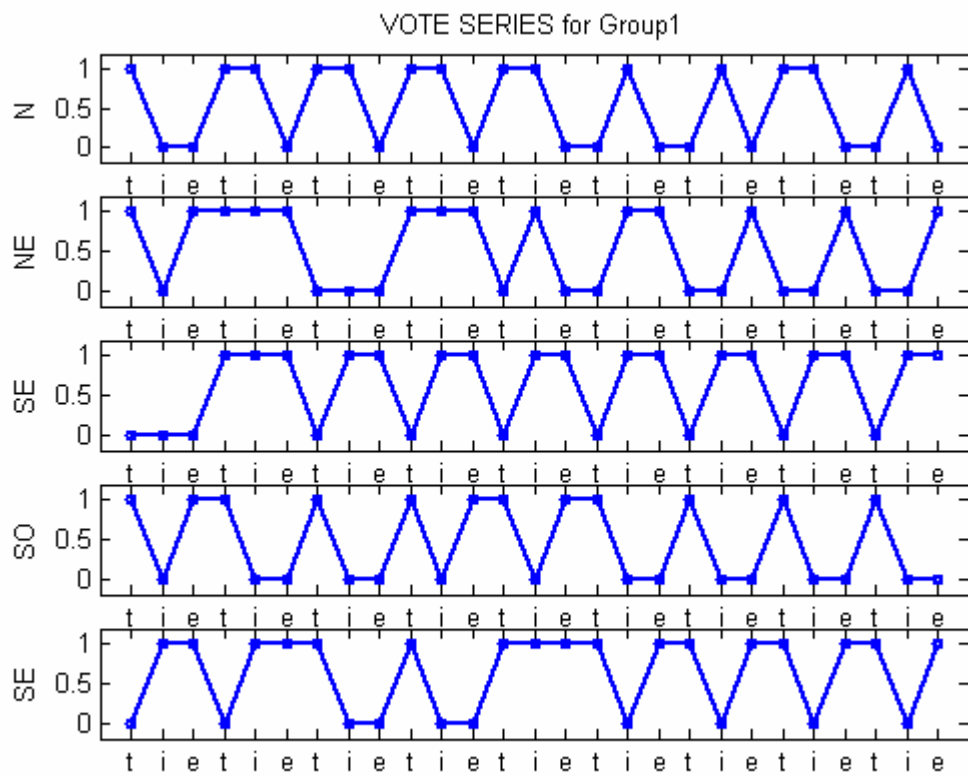


Fig. 3.2 Vote series in exp2

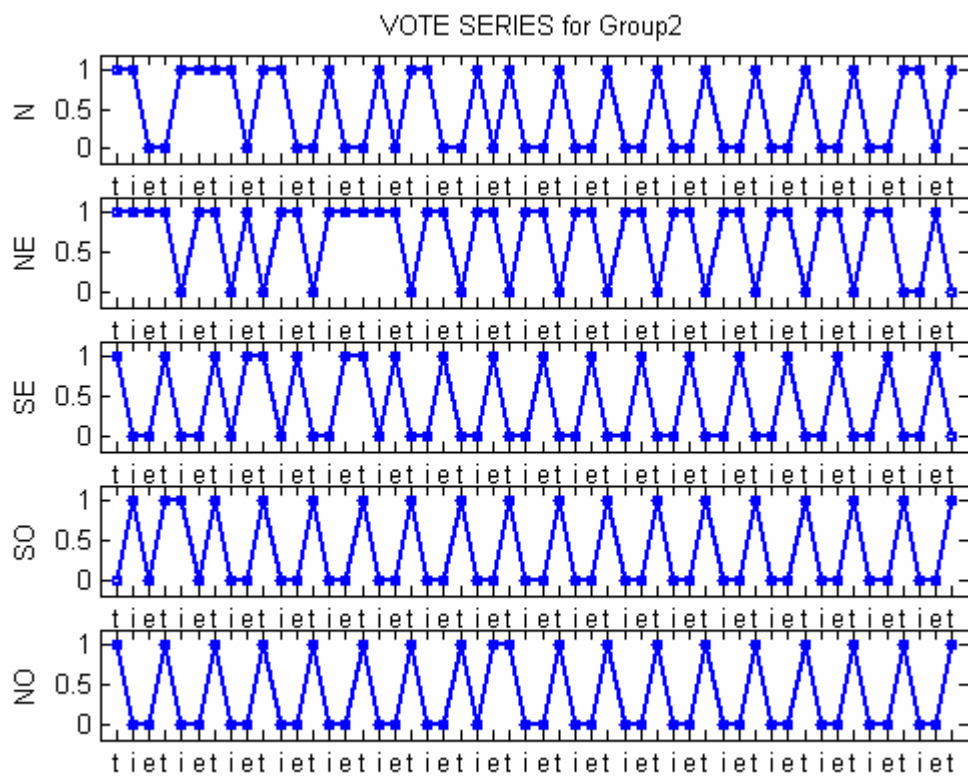


Fig. 3.3 Vote series in exp2

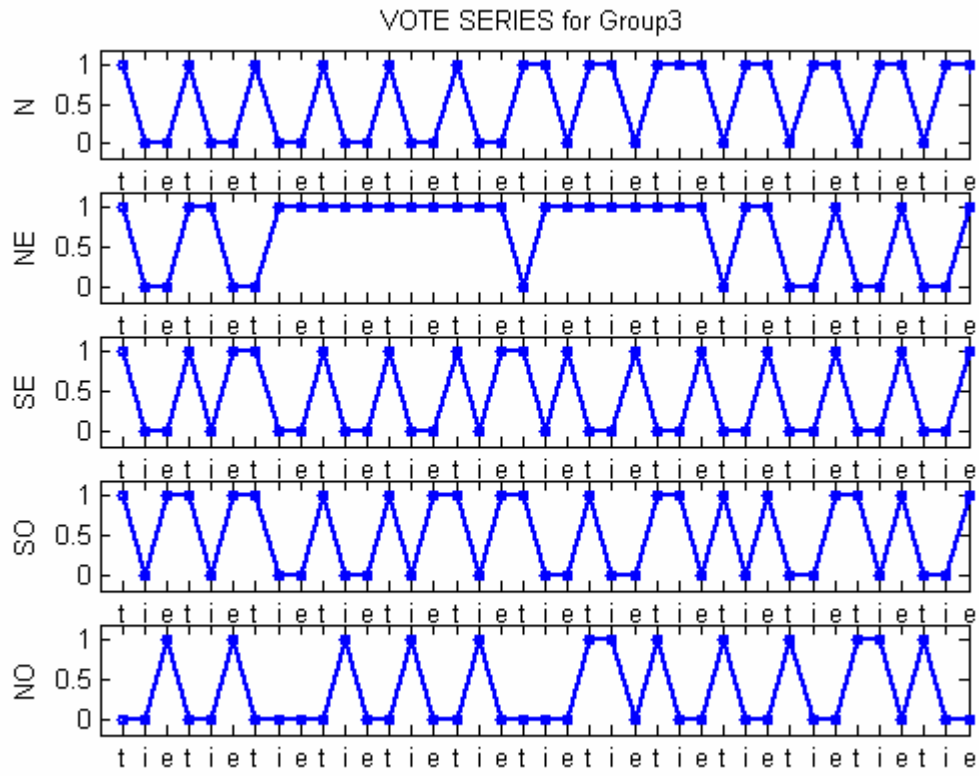
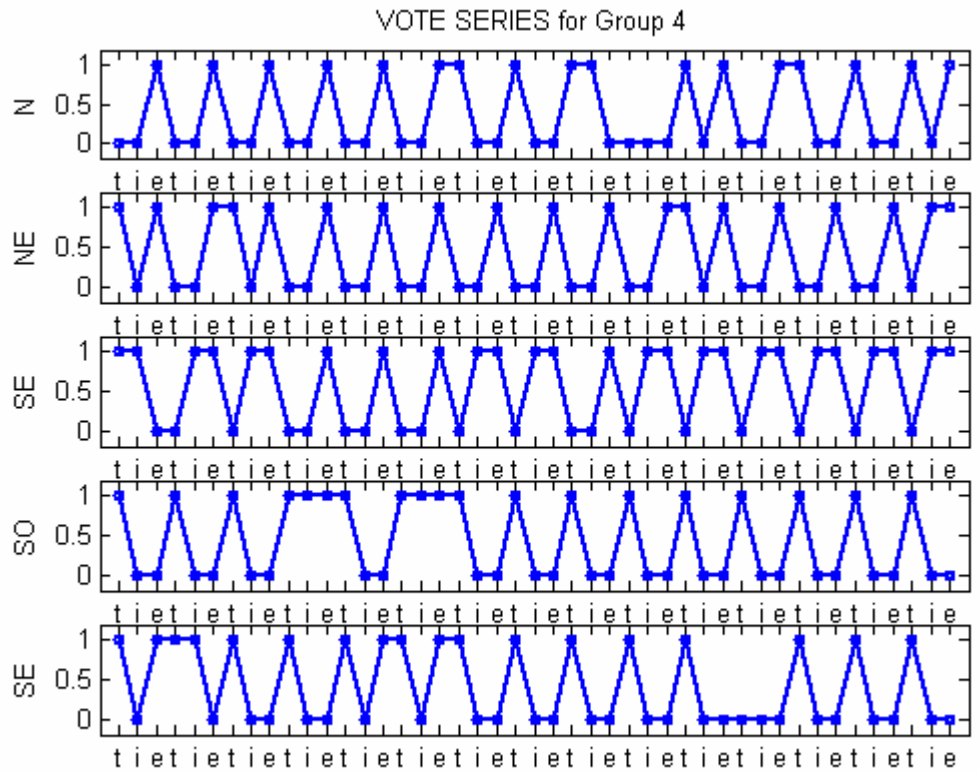


Fig. 3.4 Vote series in exp2



Tab. 3.3 Efficiency of vote (exp1)

Role	Group	Vote% T	Vote% I	Vote% E
N	1	0,00	100,00	25,00
NE	1	60,00	0,00	50,00
SE	1	0,00	0,00	0,00
SO	1	100,00	50,00	100,00
NO	1	100,00	75,00	50,00
N	2	100,00	100,00	0,00
NE	2	80,00	0,00	75,00
SE	2	80,00	0,00	100,00
SO	2	80,00	75,00	25,00
NO	2	100,00	100,00	25,00
N	3	100,00	20,00	20,00
NE	3	100,00	40,00	80,00
SE	3	20,00	0,00	100,00
SO	3	100,00	60,00	0,00
NO	3	80,00	100,00	20,00
N	4	80,00	60,00	0,00
NE	4	80,00	20,00	100,00
SE	4	80,00	0,00	100,00
SO	4	100,00	80,00	0,00
NO	4	100,00	100,00	0,00

Tab. 3.4 Efficiency of vote and ranking of rule achieved by survey (exp2)

Role	Group	Vote% T	Vote% I	Vote% E
N	1	66,67	88,89	0,00
NE	1	33,33	44,44	77,78
SE	1	11,11	88,89	88,89
SO	1	100,00	0,00	33,33
NO	1	77,78	33,33	77,78
N	2	77,78	52,94	5,88
NE	2	22,22	82,35	100,00
SE	2	27,78	0,00	88,24
SO	2	94,44	11,76	0,00
NO	2	100,00	0,00	5,88
N	3	69,23	53,85	38,46
NE	3	53,85	69,23	84,62
SE	3	53,85	0,00	69,23
SO	3	84,62	7,69	61,54
NO	3	46,15	38,46	15,38
N	4	46,67	6,67	60,00
NE	4	20,00	6,67	100,00
SE	4	6,67	73,33	93,33
SO	4	100,00	13,33	13,33
NO	4	93,33	6,67	20,00

Role	Group	Rank T	Rank I	Rank E
N	1	3	1	2(+)
NE	1	3	2(-)	1
SE	1	3	2(+)	1
SO	1	1	3	2(-)
NO	1	1	3	2(=)
N	2	3	1	2(+)
NE	2	3	2(-)	1
SE	2	3	2(+)	1
SO	2	1	2(=)	3
NO	2	1	2(+)	3
N	3	3	1	1
NE	3	3	2(-)	1
SE	3	2(+)	3	1
SO	3	2(+)	3	1
NO	3	1	2(-)	3
N	4	2(+)	3	1
NE	4	3	2(-)	1
SE	4	3	2(+)	1
SO	4	1	3	2(-)
NO	4	1	3	2(-)

Fig. 3.5: NORTH player vote efficiency (exp. 1)

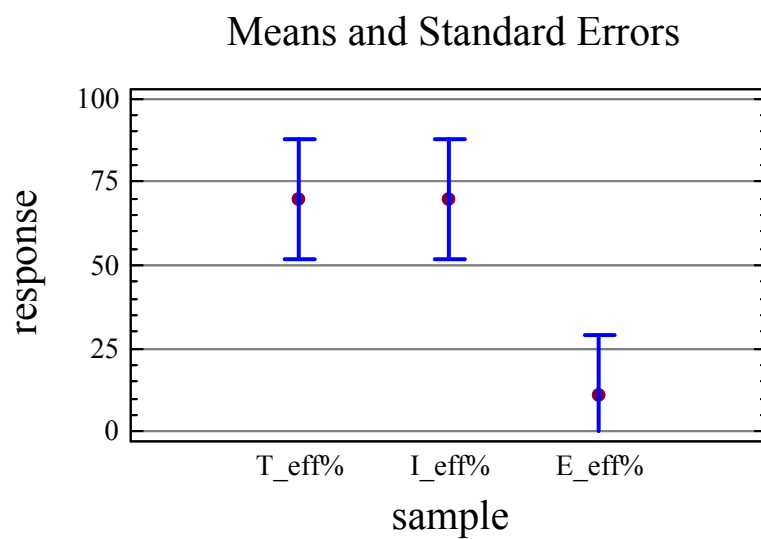


Fig. 3.6: NORTH-EAST player vote efficiency (exp. 1)

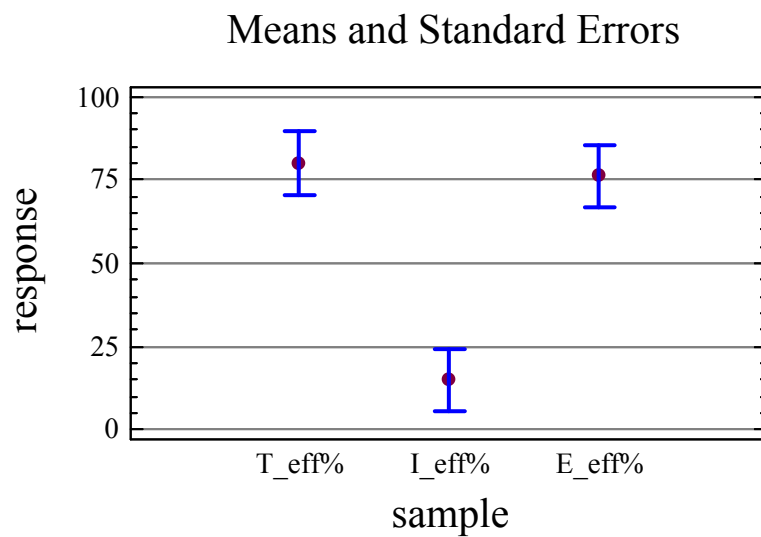


Fig. 3.7 SOUTH-EAST player vote efficiency (exp. 1)

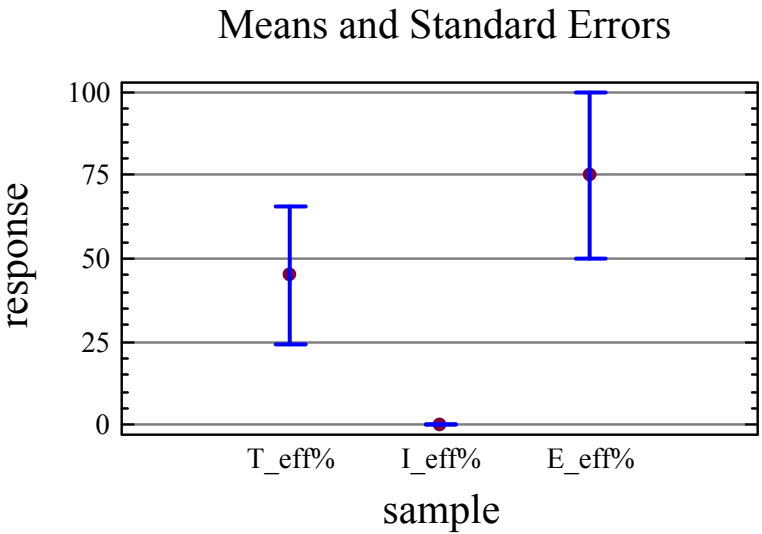


Fig. 3.8: SOUTH-WEST player vote efficiency (exp. 1)

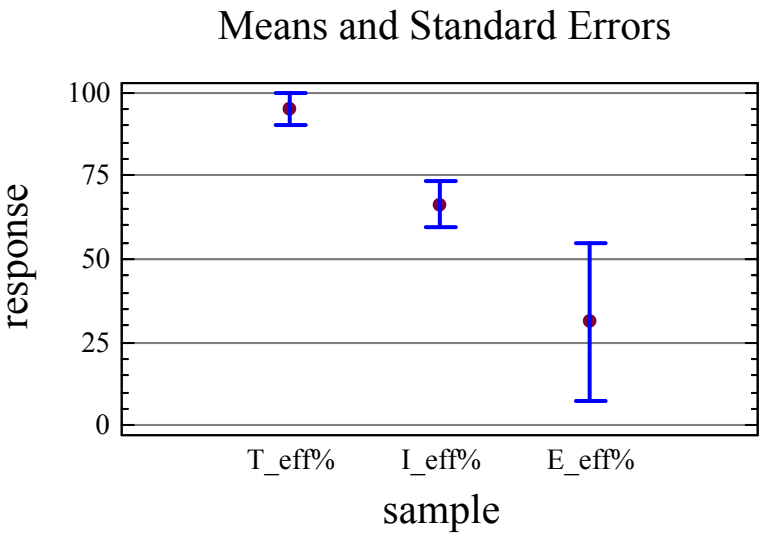


Fig. 3.9: NORTH-WEST player vote efficiency (exp. 1)

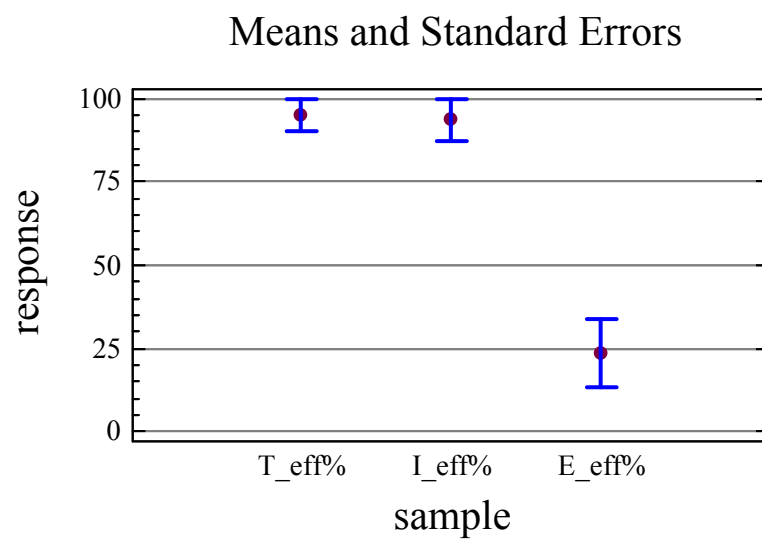


Fig. 3.10: NORTH player vote efficiency (exp. 2)

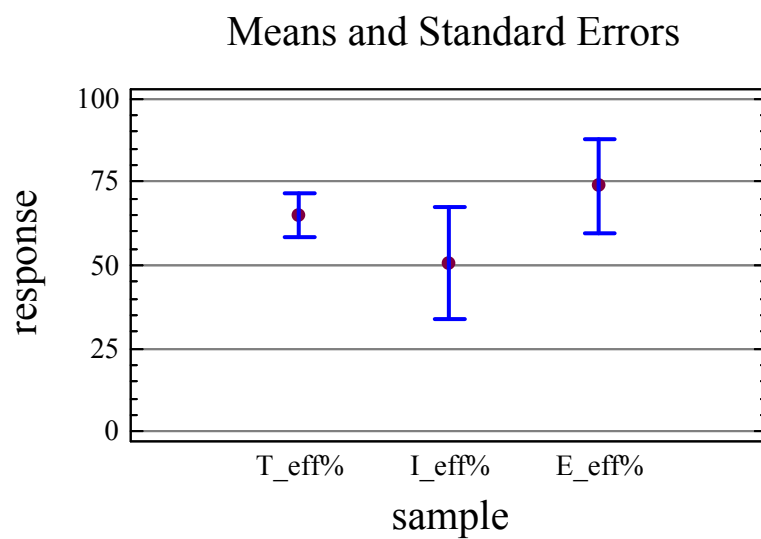


Fig. 3.11: NORTH-EAST player vote efficiency (exp. 2)

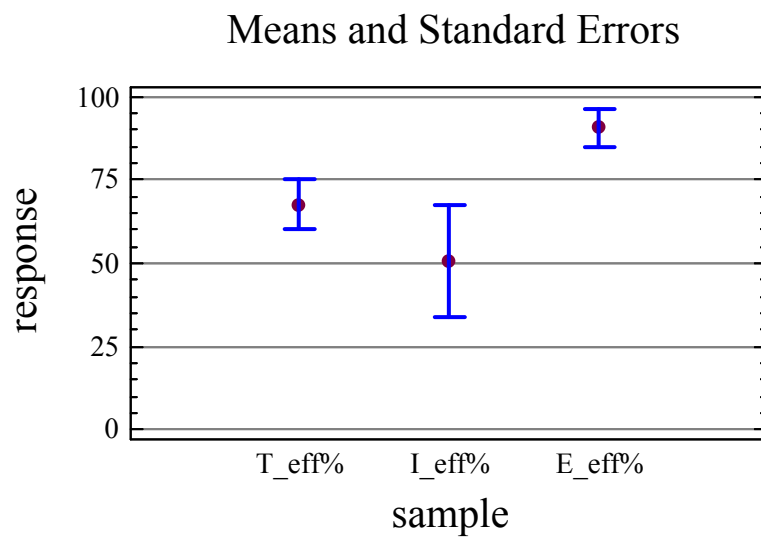


Fig. 3.12: SOUTH-EAST player vote efficiency (exp. 2)

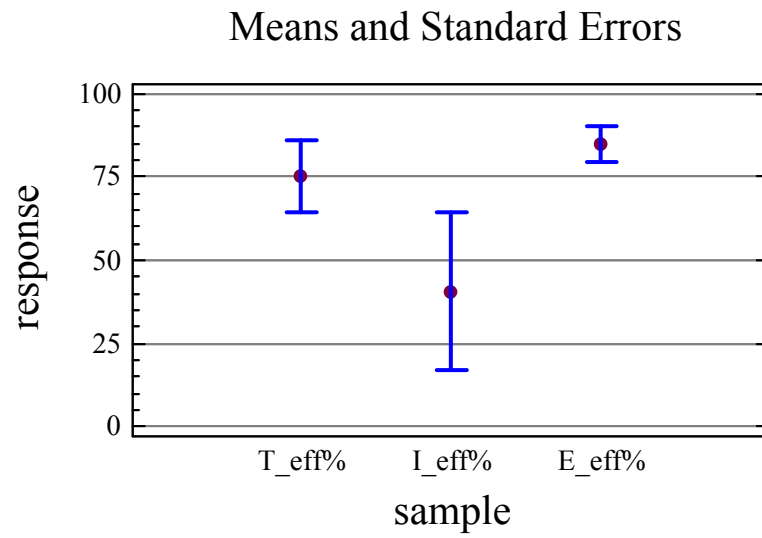


Fig. 3.13: SOUTH-WEST player vote efficiency (exp. 2)

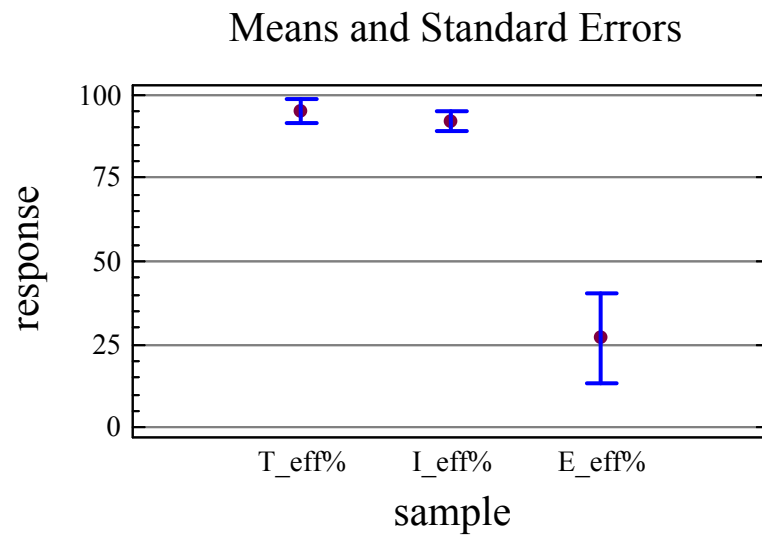
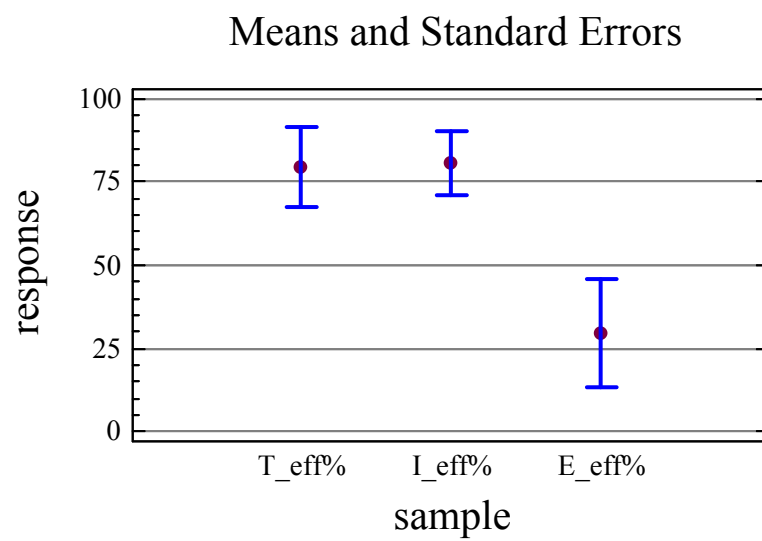


Fig. 3.14: NORTH-WEST player vote efficiency (exp. 2)



Tab. 3.5 Game behaviour and rationality (exp1)

Group 1

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	I	T	E	T	E	T	E	T
NE	m	i	e	e	i	e	m	e	m
SE	m	i	e	e	m	e	m	e	m
SO	e	i	m	e	m	e	e	e	e
NO	i	i	i	e	e	e	m	e	i
N	m	i	e	e	m	e	e	e	e

Group 2

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	E	T	E	T	E	T	E	T
NE	i	e	e	e	e	e	e	e	e
SE	e	e	m	e	e	e	i	e	m
SO	e	e	i	e	m	e	e	e	e
NO	m	e	i	e	i	e	m	e	i
N	i	e	e	e	m	e	m	e	m

Group 3

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	E	T	E	T	E	T	T	E
NE	e	e	m	e	e	e	e	i	e
SE	m	e	e	e	e	e	e	i	e
SO	m	e	m	e	m	e	m	e	e
NO	i	e	i	e	i	e	i	m	e
N	e	e	m	e	m	e	i	e	e

Group 4

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	E	E	T	E	T	E	T	E
NE	e	e	e	i	e	e	e	e	e
SE	e	e	e	e	e	e	e	e	e
SO	m	e	e	e	e	i	e	m	e
NO	i	e	e	m	e	e	e	i	e
N	e	e	e	i	e	e	e	m	e

Tab. 3.6 Game behaviour and rationality (exp2)

Group 1

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	E	T	I	E	T	T	I	E	T	I	E	I	E	E	T	E	E
NE	e	e	m	i	e	m	e	i	e	e	i	e	i	e	e	i	e	e
SE	i	e	m	i	e	m	i	i	e	i	i	e	i	e	e	m	e	e
SO	e	e	e	i	e	e	e	i	e	e	i	e	i	e	e	m	e	e
NO	e	e	i	i	e	i	m	i	e	m	i	e	i	e	e	e	e	e
N	e	e	e	i	e	e	e	i	e	e	i	e	i	e	e	e	e	e

Group 2

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	I	T	T	T	T	T	E	T	T	T	T	T	T	T	T	T	T
NE	e	i	m	e	e	e	m	e	e	e	i	e	i	e	e	m	e	e
SE	e	i	e	e	e	m	e	e	e	e	i	m	i	m	e	m	i	i
SO	e	i	e	m	m	e	e	e	m	m	e	e	e	e	m	e	e	e
NO	m	i	i	i	i	i	i	e	i	i	m	i	m	i	i	i	m	m
N	e	i	m	m	i	i	i	e	m	m	e	m	e	m	i	e	m	m

Group 3

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	T	E	T	T	T	T	E	T	T	I	T	I	E	E	E	E	E
NE	e	e	e	m	e	i	m	e	m	e	i	e	i	e	e	e	e	e
SE	e	m	e	e	e	i	m	e	e	e	i	e	i	e	e	e	e	e
SO	i	e	e	e	i	m	e	e	i	m	i	m	i	e	e	e	e	e
NO	m	i	e	i	m	e	i	e	i	i	i	i	i	e	e	e	e	e
N	i	m	e	m	m	e	e	e	e	m	i	m	i	e	e	e	e	e

Group 4

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	E	E	T	E	E	E	E	T	T	T	T	E	E	T	T	T	E
NE	m	e	e	e	e	e	e	e	e	i	i	e	e	e	m	i	e	e
SE	m	e	e	e	e	e	e	e	e	i	i	m	e	e	m	i	m	e
SO	e	e	e	m	e	e	e	e	m	e	e	m	e	e	e	e	m	e
NO	i	e	e	i	e	e	e	e	i	m	m	m	e	e	i	m	i	e
N	e	e	e	m	e	e	e	e	m	e	e	e	e	e	e	e	e	e

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