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MICROWAVE IMAGING TECHNIQUES FOR NDE/NDT WITH THE
SMW UPDATING FORMULA

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Abstract

This paper presents an innovative microwave imaging technique suitable for NDT/NDE applications. To reduce the number of problem unknowns as well as the computational load by improving the convergence rate of the process, the available *a-priori* information is exploited by introducing a computationally-effective procedure for the prediction of the electric field based on the updating Sherman-Morrison-Woodbury (SMW) formula. The numerical results as well as comparisons with state-of-the-art methods confirm the effectiveness, the feasibility, and the robustness of the proposed approach.

Introduction

Microwave imaging of dielectric objects is an expensive computational process because of the great number of unknowns and some negative features of the arising inverse scattering problem. However, testing an object for evaluating the presence of a defect allows one to reduce the computational complexity by exploiting the available *a-priori* information on the scenario under test [1]. As far as numerical inverse scattering techniques are concerned, such a problem has been faced by parameterizing the crack in order to reduce the dimension of the search space [2]-[4]. More recently, to further exploit the amount of available information on the unperturbed geometry, an improved approach has been presented in [5]. In this case, the investigation domain has been limited to the area of the unknown defect, which belongs to an inhomogeneous space (i.e., the host medium without the defect and the external background). However, although the number of unknowns has been decreased since the “investigation domain” was reduced to the area of the defect, the approach still requires the electric field prediction. Moreover, the relation between induced electric field and features of the crack is neglected (or not fully exploited). To overcome such a deficiency, this paper describes a new approach aimed at estimating in a faster and more effective fashion the electric field distribution in the investigation domain starting from the geometric features of the defect as retrieved during the iterative reconstruction process. The arising reduction of the number of unknowns leads to an increasing of the convergence rate as well as to an enhancement of the reconstruction accuracy of the inversion procedure.

Mathematical Formulation

Let us consider the two-dimensional geometry shown in Fig. 1 where the structure under test belongs to a region called investigation domain, D_{inv} , the background being a homogeneous external medium. The investigation domain is illuminated by V sources, which radiate known incident electric fields E_{inc}^v , $v=1, \dots, V$ at the working frequency f_0 . The scattered electric field E_{scatt}^v , resulting from the interactions between scatterers and fields, is collected in a set of M receivers located at a set of measurement points in an observation domain D_{obs} external to the investigation domain. Concerning the illumination, let us assume a unit plane wave TM-polarized.

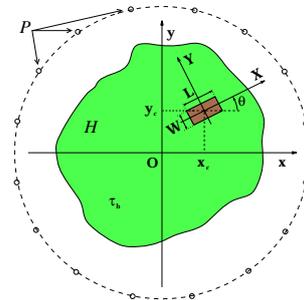


Figure 1. Problem Geometry

Mathematically, after discretization with the Richmond's method [7], the electromagnetic problem can be described as follows:

$$[E_{scatt}^v] = [G_{ext}] [\tau] [E_{tot}^v] \quad (3)$$

$$[E_{inc}^v] = [E_{tot}^v] - [G_{int}] [\tau] [E_{tot}^v] \quad (4)$$

where $[G_{ext}]$ and $[G_{int}]$ are the external and the internal Green's matrix, respectively; $[\tau]$ is a diagonal matrix whose elements are given by $\tau_{ns} = \delta_{ns} \tau(x_n, y_n)$, $\delta_{ns} = 1$ if $n=s$ and $\delta_{ns} = 0$ otherwise, $\tau(x, y)$ being the object function describing the dielectric properties of the investigation domain.

By neglecting the *a-priori* information on the scenario under test, the problem is that of determining the object function $\tau(x_n, y_n)$ and the electric field $E_{tot}^v(x_n, y_n)$ at each pixel ($n=1, \dots, N$) of D_{inv} . However, if it is assumed to approximate the defect as a homogeneous “object” of rectangular shape centered at (x_0, y_0) , characterized by a length L , a width W , and an orientation θ (see Fig. 1), then the unknown object function $\tau(x_n, y_n)$ can be expressed as a function of the features of the crack $\underline{\psi} = \{x_0, y_0, L, W, \theta\}$ as well as $E_{tot}^v(x_n, y_n)$, $n=1, \dots, N$, [through the scattering relations (3) and (4)]. The arising problem is then recast as an optimization one by defining a suitable cost function

$$\Theta(\underline{\psi}) = \frac{\sum_{v=1}^V \sum_{m=1}^M |E_{scatt}^{v,m} - \Theta_{data}^{vm}(\underline{\psi})|^2}{\sum_{v=1}^V \sum_{m=1}^M |E_{scatt}^{v,m}|^2} + \frac{\sum_{v=1}^V \sum_{n=1}^N |E_{inc}^{v,n} - \Theta_{state}^{vn}(\underline{\psi})|^2}{\sum_{v=1}^V \sum_{m=1}^M |E_{inc(U)}^{v,m}|^2} \quad (5)$$

where $E_{scatt}^{v,m}$ is the scattered electric field measured at (x_m, y_m) , $E_{inc}^{v,n}$ is the incident field in (x_n, y_n) ; $\Theta_{data}^{vm}(\underline{\psi})$ and $\Theta_{state}^{vn}(\underline{\psi})$ are the “data term” and the “state term” related to (3) and (4), respectively. Due to the non-linearity of (5) and thanks to the reduced number of problem unknowns, the minimization of $\Theta(\underline{\psi})$ is carried out by means of a suitable version of a Genetic Algorithm (GA) [8][9] according to the implementation described in [2].

As far as the computation of the unknown electric field ($[E_{tot}^v] = \mathfrak{z}(\underline{\psi})$) is concerned, a method based on the SMW formula for matrix inversion [10] is taken into account. Let us assume that the perturbed geometry (i.e., the host medium with the defect) consists of P ($P < N$) discretization sub-domains different from a known reference configuration (characterized by a known distribution $\tau_{(ref)}(x_n, y_n)$, $n=1, \dots, N$). Moreover, let us introduce the matrices, $[\Omega]$ and $[Y]$ concerned with the perturbed and the reference geometry, and given by

$$[\Omega] = \{[I] - [G_{int}] [\tau]\} \quad (6)$$

$$[Y] = \{[I] - [G_{int}] [\tau_{ref}]\}. \quad (7)$$

It should be pointed out that the unknown electric field distribution in D_{inv} can be directly computed by using a computationally expensive matrix inversion as $[E_{tot}^v] = [\Omega]^{-1} [E_{inc}^v]$. While, the computational cost strongly reduces, if $[Y]^{-1}$ is available and when the SMW updating formula

$$[\Omega]^{-1} = [Y]^{-1} + [Y]^{-1} [E] \{ [I] - [F]^T [Y]^{-1} [E] \}^{-1} [F]^T [Y]^{-1} \quad (8)$$

is used. Within such a framework, two different strategies can be adopted for the iterative estimation of $[E_{tot}^v]$. The main difference lies in the choice of the reference configuration for the dielectric profile in the investigation domain.

A. SMW Unperturbed Configuration (SMW_U)

In this case, the unperturbed geometry is assumed as the reference model, then

$$\tau_{(ref)}(x_n, y_n) = \tau_{(U)}(x_n, y_n) \quad n = 1, \dots, N \quad (9)$$

Thus, the reference matrix $[Y]$ can be computed off-line and only once during the initialization phase of the iterative minimization ($k=1$, k being the iteration index).

B. SMW Best Individual (SMW_B)

Since a multiple-agents GA-based method is used to minimize (5), the reference model is chosen at each iteration as follows

$$[\tau_{(ref)}] = [\tau]_{opt,k-1} \quad (10)$$

where $[\tau]_{opt,k-1}$ is a function of $\psi_{opt,k-1} = \arg\min_q \left(\Theta[\psi_{opt,k-1}] \right)$ according to the mapping between crack features and the profile of the object function in D_{ind} .

Numerical Validation

In this section, the SMW-based approach will be assessed by considering a selected set of numerical simulations. Both the proposed versions of the approach will be analyzed and the achieved results will be compared with those of previous implementations, (denoted by FGA [2] and IGA [5]) in terms of “location error” δ_c , the “dimensioning error” δ_a (as defined in [2]).

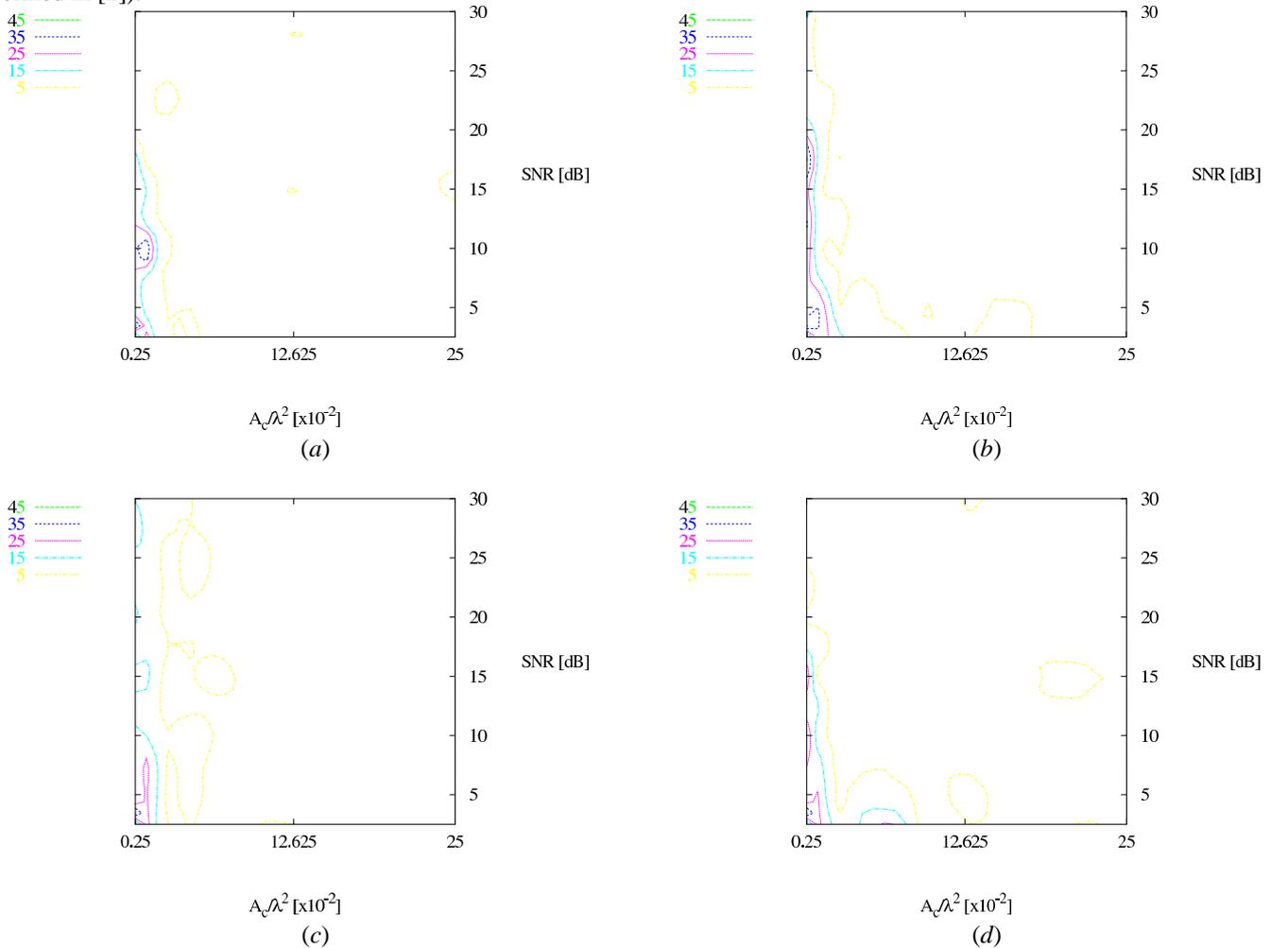


Figure 2. Behavior of δ_c versus the dimension of the defect and the SNR when (a) the SMW_U, (b) the SMW_B, (c) the FGA, and (d) the IGA are used.

Concerning the problem geometry, a square homogeneous cylinder of side $l=0.8\lambda_0$ has been illuminated by a set of $V=4$ plane-waves and the scattering data have been collected in $M=50$ measurement points equally-spaced on a circle $r=0.64\lambda_0$ in radius. Moreover, D_{inv} has been discretized into $N=256$ square sub-domains $l_{cell}=0.05\lambda_0$ -sided.

Moreover, to simulate realistic environmental conditions, a noise of Gaussian-type and characterized by a fixed signal-to-noise ratio (*SNR*) has been added to the scattering data.

The first example is aimed at evaluating the performance of the proposed approach for different *SNRs* and crack dimensions. To this end, the dimension of the defect have been varied from $A_c = 2.50 \times 10^{-3} \lambda_0^2$ up to $A_c = 2.50 \times 10^{-1} \lambda_0^2$ and the values of *SNR* in the range between 2.5 dB and 30 dB .

Figure 2 shows the contour-level representation of the localization error δ_c when the SMW_U [Figure 2(a)] and the SMW_B [Figure 2(b)] are used. For completeness, the results obtained with the FGA [Figure 2(c)] and the IGA [Figure 2(d)] are reported, as well. From such plots, it can be argued that the SMW -based approaches generally outperform the FGA procedure when $A_c \leq 12.25 \times 10^{-2} \lambda_0^2$ and the IGA technique when $A_c \geq 6.25 \times 10^{-2} \lambda_0^2$. The localization accuracy of the SMW -based techniques is further confirmed on average since $\langle \delta_c \rangle_{SMW_U} = 2.76$ and $\langle \delta_c \rangle_{SMW_B} = 3.62$ versus $\langle \delta_c \rangle_{FGA} = 3.82$ and $\langle \delta_c \rangle_{IGA} = 3.84$. Moreover, by comparing Figure 2(a) and Fig. 2(b), it turns out that SMW_B is more sensitive to the noise level than SMW_U , especially for lowest values of A_c .

As far as the dimensioning of the defect is concerned, the SMW_U slightly outperforms the SMW_B and the IGA approaches ($\langle \delta_a \rangle_{SMW_U} = 13.94$ and $\langle \delta_a \rangle_{SMW_B} = 15.37$, $\langle \delta_a \rangle_{IGA} = 14.69$). Such a behavior is more evident for low noise powers [Figs. 3(a)-(b)]. On the other hand, a larger improvement is achieved with respect to the FGA technique $\langle \delta_a \rangle_{FGA} = 33.00$.

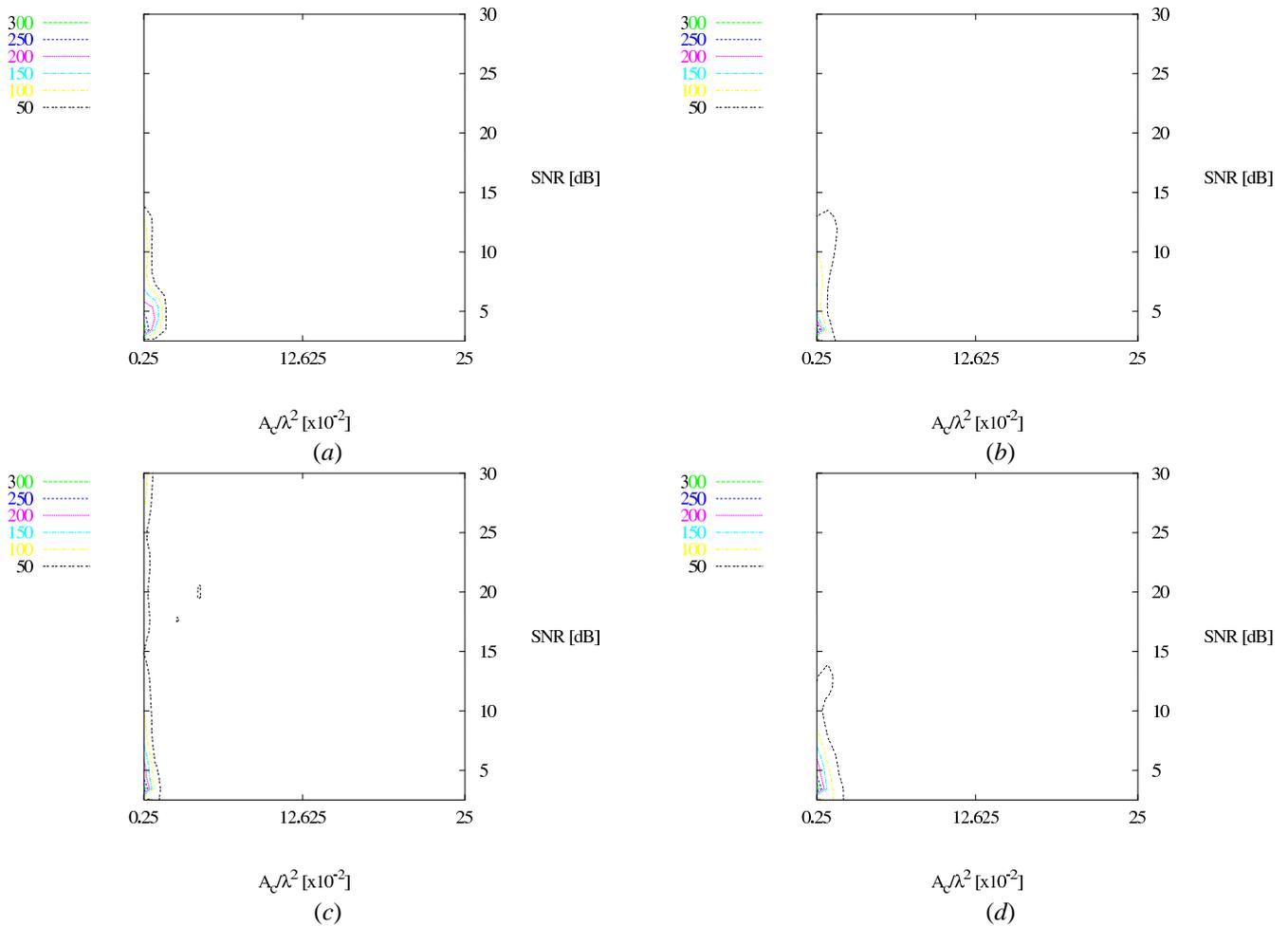


Figure 3. Behavior of δ_a versus the dimension of the defect and the *SNR* when (a) the SMW_U , (b) the SMW_B , (c) the FGA, and (d) the IGA are used.

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