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DESIGN OF NON-HARMONIC MULTI-BAND PRE-FRACTAL  
ANTENNAS

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January 2011

Technical Report # DISI-11-230



# Design of Non-Harmonic Multi-Band Pre-Fractal Antennas

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## Introduction

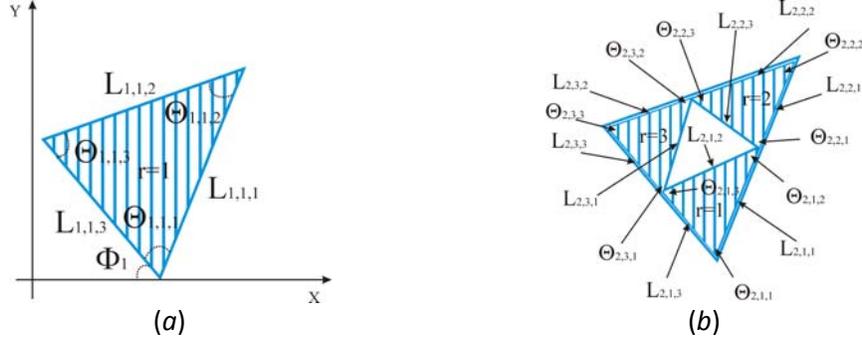
The increasing demand of electronic devices that exploit different wireless services often requires the development of customized multi-band antennas. As a consequence, the design of radiators that satisfy severe geometrical constraints and allow a multi-band behavior in the frequency spectrum is a crucial task to be carefully addressed. According to standard and general purpose approaches, the development of multi-band antennas might be addressed by considering multiple wire structures (e.g., monopoles and dipoles), each of them operating at a different working frequency or at multiple frequencies with harmonic relationships. On the other hand, another technique for enabling a multi-band behavior when dealing with wire antennas is based on suitable insertions of reactive loads on the antenna structure. Such an approach has been recently exploited by considering fractal geometries [1]. Although very attractive, this technique is intrinsically limited by the need of additional circuitual elements that could increase the complexity and costs as well as the antenna reliability. As far as fractal geometries are concerned, they are good candidates not only for miniaturization purposes [2][3], but also for the design of multi-band devices. However, rigorous fractal shapes present a harmonic-frequency rather than a multi-band behavior as pointed out in [4] for *Koch*-like fractal structures.

In this paper, an innovative fractal-based methodology is proposed for synthesizing antennas with complete and unconstrained multi-band behaviors. By assuming pre-fractal geometries as reference structures, the synthesis process is faced with an optimization procedure aimed at suitably tuning the geometrical characteristics of the antenna. In order to preliminarily assess the effectiveness and accuracy of the proposed approach, the design of a planar dual-band  $L_1$  GPS ( $f_{L_1} = 1575,42$  MHz) and *Wi-Fi* ( $f_{W-F} = 2440$  MHz) antenna is described and discussed.

## Pre-Fractal Multi-band Antenna Synthesis

The design of a non-harmonic multi-band antenna is recast as the solution of an optimization problem by defining and imposing suitable constraints on dimensions and electrical requirements (e.g., gain values,  $G_{\min}$ , and impedance matching at the input port of the antenna, i.e. a maximum value of the voltage-standing-wave-ratio,  $VSWR_{\max}$ ). Moreover, starting from a classical pre-fractal shape and in order to obtain a radiating structure that allows resonances at uncorrelated and project-specified frequencies, other degrees of freedom are added by allowing variations in the geometrical relationships among the fractal “*building*” elements at each level of spatial

resolution (or fractal stage/iteration). According to the notation used in [1], the template pre-fractal geometry, whose geometry and descriptive parameters are shown in Fig. 1, is generated by repeatedly applying the so-called *Hutchinson* operator on a *Sierpinski*-like reference shape [2]. The antenna structure at the  $i$ -th fractal stage is uniquely determined by: (a) the angle  $\Phi_1$  [Fig. 1(a)], (b) a set of segments  $L_{i,j,k}$ ,  $i=1,\dots,I$ ,  $j=1,\dots,J$ ,  $k=1,\dots,K$  in lengths; (c) a set of angles  $\Theta_{i,j,k}$ ,  $j$  ( $J=3^{i-1}$ ) being the index of the  $j$ -th triangular shape at the  $i$ -th stage and  $k$  ( $K=3$ ) denotes the  $k$ -th segment or angle belonging to the  $j$ -th triangular shape.



**Figure 1.** Descriptive parameters of the perturbed *Sierpinski*-like pre-fractal antenna: (a)  $i = 1$  and (b)  $i = 2$ .

In order to satisfy the project guidelines, the values of the descriptive parameters are determined by minimizing the following cost function

$$\Omega(\underline{\gamma}) = \min[\varphi(\underline{\gamma})] \quad (1)$$

where

$$\varphi(\underline{\gamma}) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \sum_{t=0}^{T-1} \left\{ \max \left[ 0, \frac{G_{\min}(t\Delta\theta, n\Delta\phi, m\Delta f) - \Gamma(t\Delta\theta, n\Delta\phi, m\Delta f)}{G_{\min}(t\Delta\theta, n\Delta\phi, m\Delta f)} \right] \right\} + \sum_{v=0}^{V-1} \left\{ \max \left[ 0, \frac{\Psi(v\Delta f) - VSWR_{\max}(v\Delta f)}{VSWR_{\max}(v\Delta f)} \right] \right\} \quad (2)$$

where  $\underline{\gamma} = \{\Phi_1, L_{i,j,k}, \Theta_{i,j,k}; i=1,\dots,I; j=1,\dots,J; k=1,\dots,K\}$ ,  $\Gamma(\underline{\gamma}) = \Gamma(t\Delta\theta, n\Delta\phi, m\Delta f)$  is the gain evaluated at  $\theta = t\Delta\theta$ ,  $\phi = n\Delta\phi$ ,  $f = m\Delta f$  and  $\Psi(\underline{\gamma}) = \Psi(v\Delta f)$  is the value of the *VSWR* computed at  $f = m\Delta f$ . Moreover,  $\Delta f$  is the sampling frequency step,  $\Delta\theta$  and  $\Delta\phi$  are the angular steps in elevation and azimuth direction.

### Numerical and experimental results

With reference to the synthesis of a  $L_1$ -GPS and  $Wi-Fi$  dual-band antenna, the following constraints has been imposed: (i)  $VSWR \leq VSWR_{\max} = 2.0$ ; (ii) gain values greater than  $G_{\min} = 3.0dB_i$  at  $\theta = 0^\circ$  and greater than  $G_{\min} = -4.0dB_i$  at  $\theta = 70^\circ$  in the

$L_1$ -GPS band by taking into account the presence of a 20 dB preamplifier stage; (iii) an hemispherical coverage in the  $Wi-Fi$  band. Concerning its physical dimension, the antenna is required to belong to a square physical platform ( $\lambda_{L1}/4 \times \lambda_{L1}/4$ ) (iv).

In order to synthesize such a dual-band antenna, a two-order ( $I = 2$ ) pre-fractal shape has been adopted and tuned by minimizing the cost function in (1) with a suitable implementation [5] of the Particle Swarm Optimizer (PSO) [6] integrated with a Method-of-Moments (MoM) electromagnetic simulator for carefully modeling the presence of the dielectric slab and of a reference ground plane.

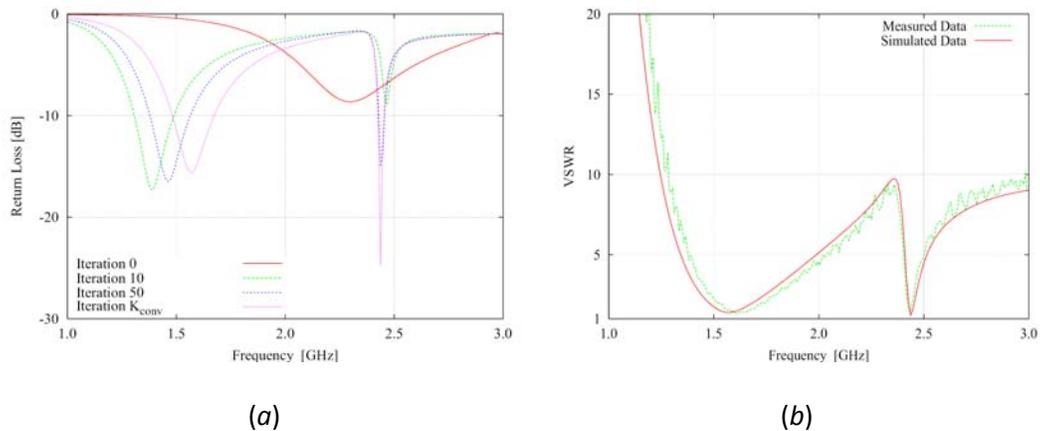
The design shown in Fig. 2(a), with dimensions equal to 28 mm along the  $y$ -axis and 32 mm along the  $x$ -axis, has been synthesized.



**Figure 2.** Dual-band Sierpinski-like pre-fractal antenna: (a) geometry of the synthesized antenna and (b) antenna prototype.

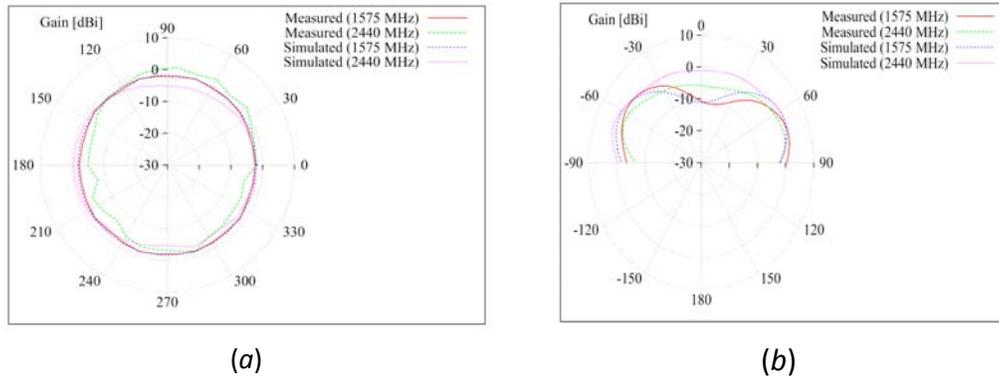
The optimization required  $K_{conv} = 250$  iterations and an amount of CPU-time of about 10 sec at each iteration (Pentium IV 1800 MHz, 512 MB RAM).

Figure 3(a) shows the evolution of the plot of the Return Loss during the iterative optimization process. Moreover, simulated and experimentally-measured values (in the presence of a metallic ground plane of dimension  $90 \times 140 \text{ cm}^2$ ) of the VSWR at the input port of the antenna are compared in Fig. 3(b). As it can be noticed, there is a good agreement between simulated data and measurements (GPS- $L_1$  band:  $VSWR_{sim} = 1.39$  vs.  $VSWR_{meas} = 1.59 \text{ dB}$ ;  $Wi-Fi$  band:  $VSWR_{sim} = 1.22$  vs.  $VSWR_{meas} = 1.45$ ).



**Figure 3.** Dual-band Sierpinski-like pre-fractal antenna: (a) simulated values of the Return Loss and (b) measured and simulated VSWRs.

Finally, as expected, the synthesized antenna allows a hemispherical coverage in the  $Wi - Fi$  band (Fig. 4) and the gain values in the GPS-  $L_1$  band (with a 20 dB preamplifier stage) at  $\theta = 0^\circ$  and  $\theta = 70^\circ$  are greater than specifications [ $\Phi(\theta = 0^\circ) > G_{\min}(\theta = 0^\circ) = 3.0 \text{ dB}_i$  and  $\Phi(\theta = 70^\circ) > G_{\min}(\theta = 70^\circ) = -4.0 \text{ dB}_i$ ] [Fig. 4(b)].



**Figure 4.** Dual-band *Sierpinski*-like pre-fractal antenna. Gain function - Simulated and measured values: (a) horizontal plane and (b) vertical plane ( $\phi = 0^\circ$ ).

## Conclusions

A procedure for the design and optimization of non-harmonic multi-band pre-fractal antennas has been described. As a representative example, an antenna structure has been synthesized through the *PSO*-based procedure by optimizing the descriptive parameters of a “perturbed” *Sierpinski*-like pre-fractal geometry in order to comply with the requirements in the  $L_1$ -GPS and  $Wi - Fi$  frequency bands. A set of comparisons between simulated data and measurements has been carried in order to verify the reliability and efficiency of the optimized prototype as well as for drawing a preliminary assessment of the non-harmonic multi-band synthesis procedure.

## Acknowledgment

This work was partially supported in Italy by *Study and Development of Innovative Smart Systems for Highly Reconfigurable Mobile Networks*, Progetto di Ricerca di Interesse Nazionale - Miur Project COFIN 2005099984 and by *ARSLOGICA-CEMIA*, DIT-PRJ-06-090 - Industrial Research Project with *Arslogica S.r.L.*

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