SIMULTANEOUS OPTIMIZATION OF SUBARRAY WEIGHTS AND SIZES FOR LOW SIDELOBE SYNTHESIS OF LARGE ARRAY ANTENNAS

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Introduction

Large array antennas of wide dimension are often used in several communication and radar systems. Thanks to their performance (e.g., the patterns are electronically steerable, the illumination can be controlled directly on the aperture, etc.), array antennas have replaced parabolic reflectors in many challenging and real applications. Nevertheless, large phased arrays still represent too expensive solutions because of the large number of radiating elements and the high circuit complexity. For this reason, the subarraying strategy has been widely adopted in order to simplify the antenna design and to achieve a better trade-off between cost and performances. Accordingly, the elements on the array aperture are grouped into clusters and a gain is assigned to each of them. Since the use of amplitude weights at the output of the subarrays generates unavoidable grating lobes [1], various strategies have been proposed to synthesize subarrayed antennas with low sidelobe levels (SLLs) [2]-[4].

In this paper, the procedure proposed in [5] is extended to deal with such a synthesis problem. In particular, the joint optimization of the number of elements in a subarray and the corresponding subarrayed gains to achieve the lowest $SLL$ is carried out by means of an effective excitation matching procedure. The knowledge of the optimal distribution [6] of the array coefficients and the convexity of the problem at hand with respect to a subset of the unknowns are exploited to suitably define the subarrayed weights. On the other hand, they allow an enhancement of the convergence rate of the numerical procedure by means of a proper reduction of the dimension of the solution space. Selected results of a set of preliminary examples concerned with arrays of isotropic elements are reported to assess the effectiveness and to envisage the potentialities of the proposed approach.

Problem Statement and Mathematical Formulation

Let us consider a linear array antenna with $N = 2M$ elements equally-spaced of the quantity $d = \lambda/2$, $\lambda$ being the free-space wavelength. The corresponding far-field pattern is given by [6]
\[ F(\vartheta) = \sum_{n=-M}^{M} c_n e^{j2\pi nvd \cos(\vartheta)} \]  

(1)

where \( k = 2\pi/\lambda \), \( \vartheta \) is the angular location from the boresight direction, and \( c_n = \delta_{nq} w_q \), \( n = -M, \ldots, 1, \ldots, M \), being \( \delta_{nq} = 1 \) if \( a_n = q \) and \( \delta_{nq} = 0 \) otherwise. The integer value \( a_n \in [1, Q] \), \( n = \pm 1, \ldots, \pm M \), indicates the membership of each element to the corresponding subarray, \( q = 1, \ldots, Q \).

According to the subarraying strategy, the array elements are grouped into \( Q \) different and physically contiguous clusters and a gain \( w_q \), \( q = 1, \ldots, Q \), is assigned to each cluster. Moreover, since the elements on the aperture are supposed to be uniformly excited (i.e., \( g_n = 1 \), \( n = \pm 1, \ldots, \pm M \)) the subarray gains turn out to be equal to the element coefficients [4]. Thus, the synthesis problem at hand can be formulated as the definition of the subarray configuration, subject to physical contiguity constraints, and the corresponding subarray weights that minimize the maximum SLL of the compromise pattern.

Towards this end, an excitation matching approach [5] is used. In particular, the problem has been properly reformulated [7] as the minimization of the following cost function

\[ \Psi(a_n, w_q) = \sum_{q=1}^{Q} \sum_{n=-M}^{M} |\gamma_n - \delta_{nq} w_q|^2 \]  

(2)

where \( \gamma_n \), \( n = 1, \ldots, N \) are the optimal/reference excitation coefficients to match (e.g., those of either a discrete Taylor or Bayliss distribution [6]). Since the problem is convex respect to the subarray coefficients (for a given clustering) [8], the optimal value of \( w_q \) can be analytically obtained by simply nulling the first derivative of (2). Accordingly, it turns out that

\[ w_q = \frac{\sum_{n=-M}^{M} \gamma_n \delta_{nq}}{\sum_{n=-M}^{M} \delta_{nq}} \text{, } q = 1, \ldots, Q . \]  

(3)

Then, the use of the so-called Contiguous Partition Method (CPM) [5] allows: (a) an easy and profitable representation of the whole set of allowed clustering and (b) the use of an efficient resolution algorithm, namely the Border Element Method (BEM), to sample the solution space. Unlike [5], where the excitations close as much as possible to the optimal ones have been looked for, the aim is now to define the problem solution which affords a pattern with minimum SLL.
Experimental Results

In order to give an indication of the capabilities and potentialities of the proposed approach, a selected representative result is reported in the following. It is concerned with the synthesis of a \( N = 70 \) element array with \( Q = 10 \) subarrays. Concerning the optimal setup, a Taylor sum pattern with \( SLL = -30 \, dB \) and \( \bar{n} = 4 \) has been considered and computed with the Villeneuve technique [6]. Because of the symmetric behavior of the reference distribution of the element coefficients, \( Q = 5 \) subarrays have been dedicated to each half of the array as in [4]. Figure 1 shows the reference pattern together with that synthesized with the proposed approach. It is worth noting that the maximum \( SLL \) of the subarrayed solution is equal to \( -28.85 \, dB \), slightly above
the reference value. Moreover, it should be pointed out that the final solution has been found after 17 function evaluations with an overall required CPU-time of 23 sec. For completeness, the optimized number of elements assigned to each subarray as well as the corresponding subarray gains are reported in Fig. 2.

Conclusions

In this paper, the synthesis of large linear arrays by means of contiguous subarrays has been dealt with. An effective approach for the simultaneous optimization of the array clustering and the subarray gains has been presented. Essentially based on an excitation matching procedure and exploiting the convexity of the functional with respect to the subarray weights, the solution space has been properly reduced and an efficient sampling algorithm has been used to look for the “compromise” solution affording a pattern with the lowest SLL. Preliminary results have been reported in order to show potentialities and limitations of the proposed approach.

References: