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Multi-resolution Approaches for Inverse Scattering Problems

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Abstract: In the framework of microwave imaging, inverse scattering techniques currently represent the state-of-the-art for the quantitative reconstruction of the region under test. In such a framework, multi-resolution techniques have recently appeared in order to increase the effectiveness of the inversion procedure. To properly deal with the ill-posedness characterizing their mathematical model and at the same time enhance the achievable spatial resolution, such a paper presents and discusses several multi-resolution approaches based on the use of multi-resolution and suitable basis functions for the problem unknowns.

Keywords: Microwave Imaging, Inverse Scattering, Multi-resolution techniques.

1. Introduction

The detection of target buried in an inaccessible host medium plays a relevant role in the field of nondestructive evaluation and testing (NDE/NDT) of industrial artifacts [1], subsoil inspection [2], biomedical imaging [3], and through-the-wall-imaging (TWI) [4]. Because of the ability of electromagnetic fields at centimeter wavelength to penetrate a large number of materials (except for ideal conductors), microwave imaging techniques appear to be very effective compared to other state-of-the-art methods (e.g., x-rays, ultrasound, and eddy currents). As a matter of fact, they usually require low power levels and do not need a mechanical contact between object and source [5].

In order to obtain quantitative reconstructions of the domain under test, inverse scattering techniques have been profitably employed [6]-[8]. However, further efforts are still necessary to allow the feasibility in real applications. As a matter of fact, the underlying mathematical model is characterized by several drawbacks, such as ill-posedness [9] and non-linearity [10], that limit their effectiveness because of the reduced achievable spatial resolution and non-negligible computational costs. To cope with the ill-posedness, multi-view/multi-illumination systems are adopted to collect sufficient amount of data. However, the information available from the scattering experiments is upper-bounded and the number of independent data is lower than the dimension of the solution space [11][12]. Therefore, effective strategies aimed at fully exploiting the scattering data must be adopted in order to increase the effectiveness of the reconstruction process.

A possible solution consists in the use of multi-resolution strategies that provide an enhanced spatial resolution only in those regions of interest (*RoI*s) where discontinuities occur [13][14] and/or where the unknown scatterers are located [15]. Recently, adaptive multi-step approaches have been proposed to iteratively increase the spatial resolution through a "zooming" procedure [15], at the same time keeping a fixed low ratio between unknowns and data to minimize the occurrence of local minima. However, multi-resolution can be obtained by exploiting suitable basis functions, as well.

This contribution focuses on the use of multi-resolution approaches in order to enhance the quantitative and qualitative [16] reconstruction in inverse scattering problems. Towards this end, several strategies based on the exploitation of the iterative multi-scaling approach (*IMSA*) [15] and on the use of non-standard basis functions will be presented and discussed.

2. **Mathematical Formulation**

Let us consider a 2-D investigation region D_{ind} illuminated by a set of *TM* plane waves $\underline{E}_v^{inc}(\underline{r}) = E_v^{inc}(\underline{r}) \cdot \hat{\underline{z}}$, $v = 1,...,V$, $\underline{r} = (x, y)$. The total field, $E_v^{tot}(\underline{r})$, is collected at *M* measurement points $\underline{r}_m$, $m = 1,..., M$, located in a region, called observation domain D_{obs}, external to D_{ind}. The scenario under test is described by the following contrast function

$$\tau(\underline{r}) = [\varepsilon(\underline{r}) - 1] - j\frac{\sigma(\underline{r})}{2\pi f \varepsilon_0} \qquad \underline{r} \in D_{ind} \tag{1}$$

where $\varepsilon(\underline{r})$ and $\sigma(\underline{r})$ are the relative permittivity and conductivity distributions, f and ε_o being the working frequency and the background permittivity, respectively.

As regards to the scattering phenomena, the interactions between objects and fields are described by the following integral equations

$$E_v^{tot}(\underline{r}_m) = E_v^{inc}(\underline{r}_m) + \left(\tfrac{2\pi}{\lambda}\right)^2 \int_{D_{ind}} \tau(\underline{r}') E_v^{tot}(\underline{r}') G_{2D}(\underline{r}_m, \underline{r}') d\underline{r}' \quad \underline{r}_m \in D_{obs} \tag{2}$$

$$E_v^{inc}(\underline{r}) = E_v^{tot}(\underline{r}) - \left(\tfrac{2\pi}{\lambda}\right)^2 \int_{D_{ind}} \tau(\underline{r}') E_v^{tot}(\underline{r}') G_{2D}(\underline{r}, \underline{r}') d\underline{r}' \quad \underline{r} \in D_{ind} \tag{3}$$

where λ is the background wavelength and $G_{2D}(\underline{r}, \underline{r}') = -\frac{j}{4} H_0^{(2)}\left(\frac{2\pi}{\lambda}\left\|\underline{r} - \underline{r}'\right\|\right)$ is the free-space 2-D Green's function, $H_0^{(2)}$ being the second-kind 0-th order Hankel function.

Since the problem unknowns $\tau(\underline{r})$ and $E_v^{tot}(\underline{r})$ are not available as a closed-form solution, equations (2) and (3) are discretized and a numerical solution is looked for by minimizing the following cost function

$$\Theta\{\tau, E^{tot}\} = \frac{\left\|E_v^{tot}(\underline{r}_m) - f_m(\tau, E^{tot})\right\|_{D_{obs}}}{\left\|E_v^{tot}(\underline{r}_m)\right\|_{D_{obs}}} + \frac{\left\|E_v^{inc}(\underline{r}) - f(\tau, E^{tot})\right\|_{D_{ind}}}{\left\|E_v^{inc}(\underline{r})\right\|_{D_{ind}}} \tag{4}$$

where $f_m(\tau, E^{tot})$ and $f(\tau, E^{tot})$ are the reconstructed quantities [10].

In order to cope with the ill-posedness of the problem at hand, a multi-resolution procedure can be considered. A possible solution consists in the use of a multi-step sequence that enhance the spatial resolution only in the region of interest (*RoI*) where the scatterer is supposed to be located. Such a zooming procedure allows to reduce the number of unknowns during the inversion procedure, thus minimizing the risk of the occurrence of local minima [9]. On the other hand, the multi-resolution can be also obtained by using suitable sets of basis function to represent the parameters to be retrieved, so that the physical quantities can be expressed using less unknowns without decreasing the accuracy of the representation.

3. **Quantitative Multi-resolution Imaging Methods**

Quantitative imaging methods are aimed at reconstructing the contrast and the electric field in the whole investigation domain. As a consequence, the number of unknowns is usually larger than the number of independent data and the problem at hand results to be highly ill-posed. As a consequence, multi-resolution approaches have to be considered.

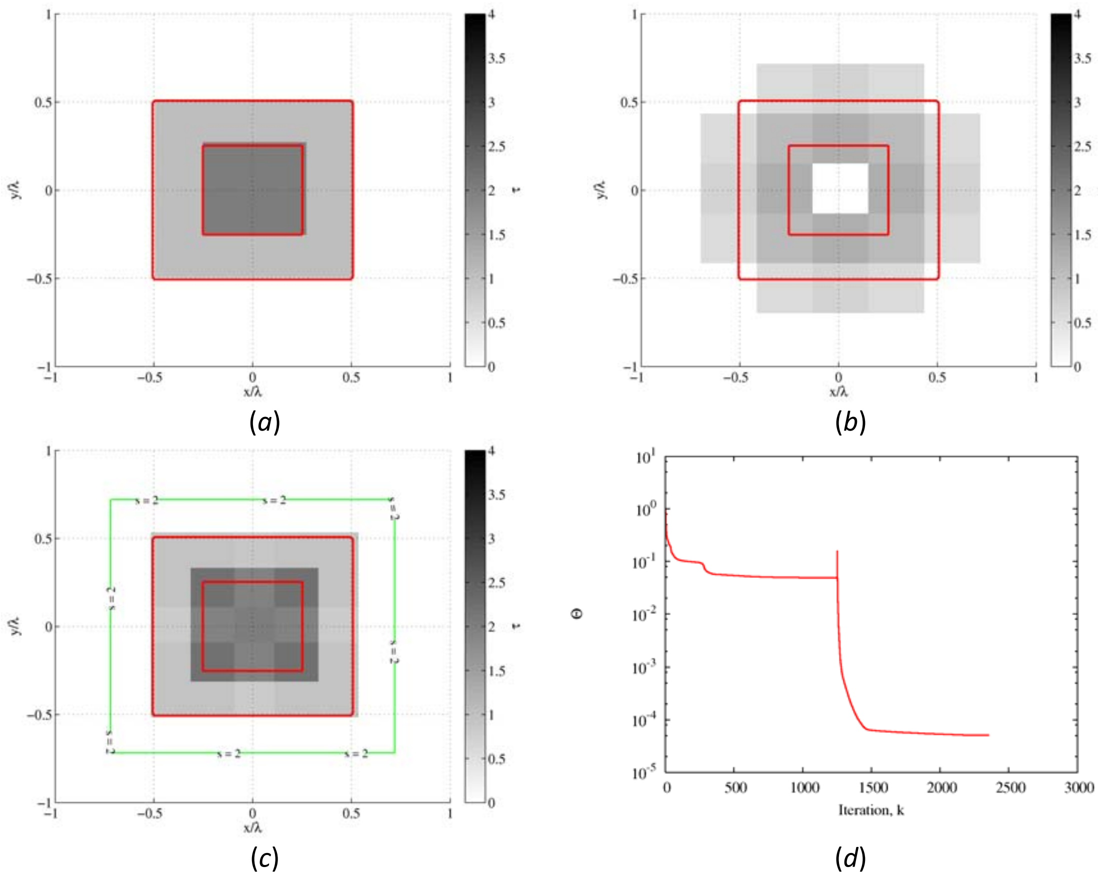

Fig. 1 – Reconstruction of a square cylinder: (*a*) actual profile and results provided by the proposed approach at (*c*) $s = 1$ and (*d*) $s = 2$. Behavior of the cost function (*d*).

Towards this end, a multi-step procedure characterized by S steps can be adopted by expressing the problem unknowns $\tau(\underline{r})$ and $E_v^{tot}(\underline{r})$ as follows

$$\tau(\underline{r}) = \sum_{s=1}^{S} \sum_{n=1}^{N} \tau_{n,s} B_{n,s}(\underline{r}) \tag{5}$$

$$E_v^{tot}(\underline{r}) = \sum_{s=1}^{S} \sum_{n=1}^{N} E_{v,n,s}^{tot} B_{n,s}(\underline{r}) \tag{6}$$

where $\tau_{n,s}$ and $E_{v,n,s}^{tot}$ are the contrast and the total field evaluated in the *n,s*-th sub-domain of the investigation domain, $D_{n,s}$, $n = 1,...,N$, $s = 1,...,S$. As a consequence, starting from an initial guess by considering the whole D_{ind} as *RoI*, a rough solution is firstly retrieved. Then, by processing such a result in order to perform a zooming procedure, the *RoI* is re-defined as the area where the scatterer is located and a new step is carried out. As a representative example, Figure 1 shows the result of the reconstruction by means of the iterative multi-scaling approach (*IMSA*) [15] of a lossless square cylinder characterized by two permittivity values, $\varepsilon_1 = 2.0$ and $\varepsilon_2 = 3.0$ [Fig. 1(*a*)]. $V = 4$ *TM* plane waves impinging from the directions $\theta_v = 2\pi(v-1)/V$, $v = 1,...,V$, have been considered and the noiseless measurements have been collected at $M = 20$ receivers uniformly distributed on a circle of radius $\rho = 2\lambda$. At each step, the region of interest has been discretized in $N = 49$ square sub-domains. The improvement of spatial resolution provided by the IMSA can be noticed from the comparison between the results of Fig. 1(*b*) and Fig. 1(*c*). Figure 1(*d*) shows the behavior of the cost function, where the error rapidly decrease when moving from $s = 1$ and $s = 2$.

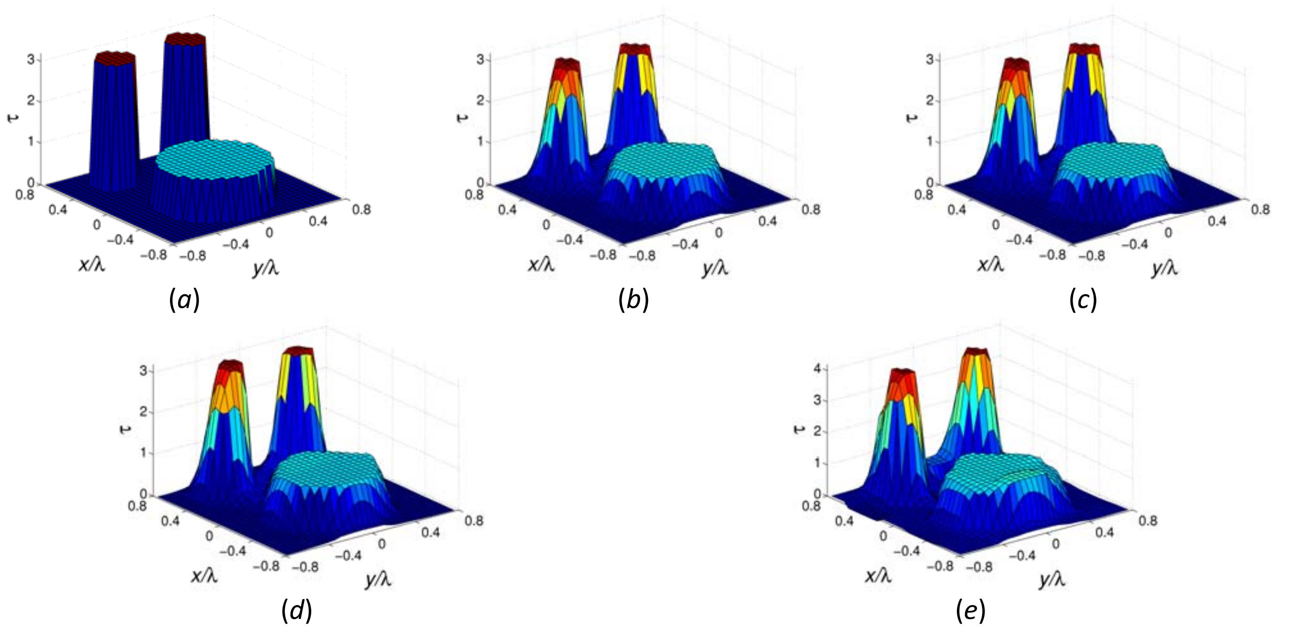

Fig. 2 – Reconstruction with different size of the wavelet basis. (*a*) Actual contrast distribution, (*b*) reconstruction with the standard method, and (*c*)(*d*)(*e*) reconstruction with wavelet basis. (*c*) *P* = 769, (*d*) P = 193, and (*e*) P = 48.

On the other hand, a multi-resolution reconstruction can be obtained also by using different sets of basis function to represent the unknowns. Accordingly, the equivalent current density $J_v(\underline{r}) = \tau(\underline{r})E_v^{tot}(\underline{r})$ can be expressed as

$$J_v(\underline{r}) = \sum_{p=1}^{P} J_{v,p} C_p(\underline{r}) \tag{7}$$

where $J_{v,p}$ is the projection of the current $J_v(\underline{r})$ on the basis $\{C_p(\underline{r}); p = 1,\ldots,P\}$. Figure 2 shows a representative example of a reconstruction of three circular cylinders characterized by different permittivity values [Fig. 2(*a*)] by means of a wavelet basis with size *P*. The results have been obtained by considering $M = V = 32$ and from data blurred by noise characterized by a signal to noise ratio (*SNR*) equal to 20 *dB*. As it can be observed, when $P = 769$ [Fig. 2(*c*)] the reconstruction appears to be very similar to the standard case ($N = 1024$) [Fig. 2(*b*)]. An equivalent reconstruction can be obtained also with a reduction of 80% of the number of unknowns ($P = 193$) [Fig. 2(*d*)]. On the contrary, when $P = 193$ [Fig. 2(*d*)], the reconstruction is only qualitatively acceptable, whereas the estimation of the contrast values is not accurate.

4. **Qualitative Multi-resolution Imaging Methods**

The multi-resolution can be exploited in order to improve the effectiveness of qualitative imaging method, as well. In such a case, the unknowns to be retrieved are the shape and the position of the targets, since the contrast values are assumed to be *a-priori* known quantities. In the framework of qualitative imaging method, level set (*LS*) method is an effective shape optimization approach based on the representation of the shape of the target by means of the zero level of a level set function ϕ defined by means of a signed distance [17].

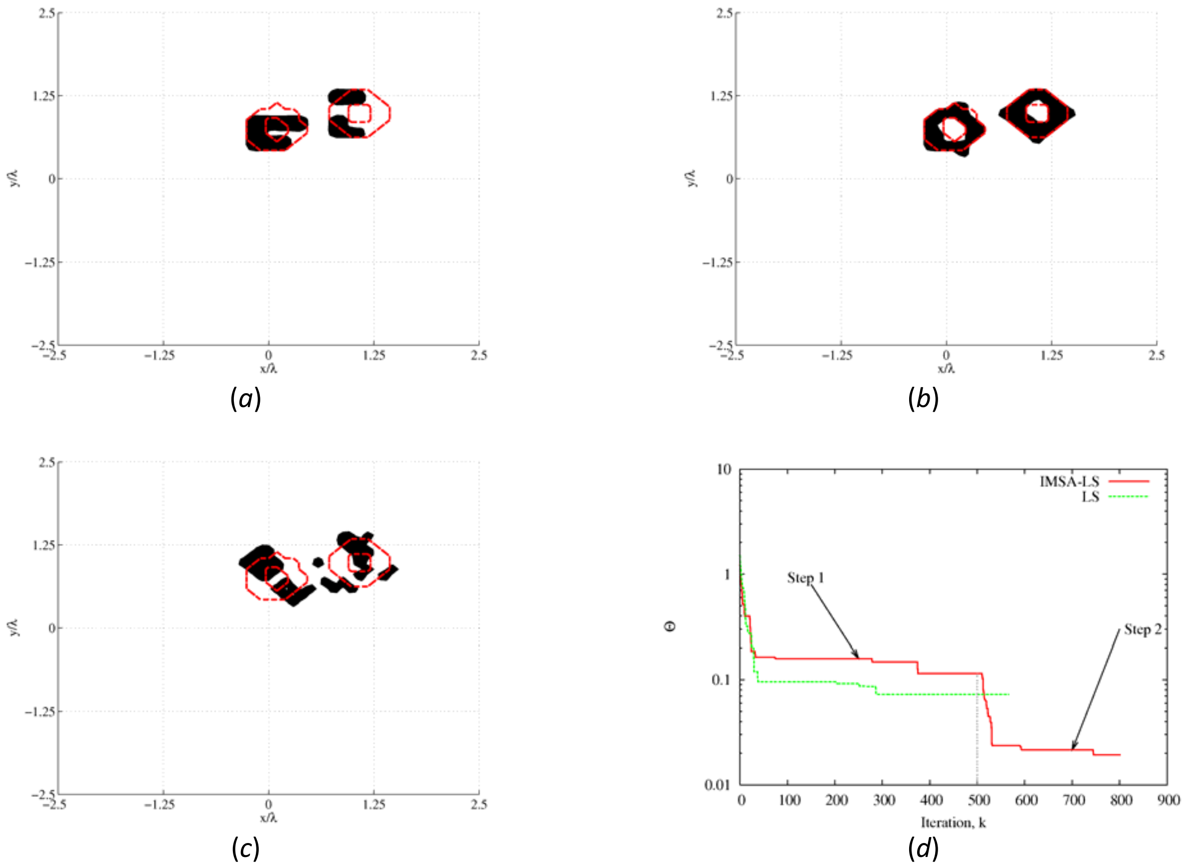

Fig. 3 – Reconstruction of two-hollow cylinders by means of (*a*) *IMSA-LS* at *s* = 1, (*b*) *IMSA-LS* at *s* = 1, and (*c*) LS. Behavior of the cost function (*d*).

The level set function is negative inside the object, positive outside and it is iteratively modified by means of a gradient-based technique. According to its value, such a function determines whether a point belongs to the trial shape or not. By considering a multi-resolution representation, ϕ can be suitably re-defined as follows

$$\phi(\underline{r}) = \sum_{s=1}^{S}\sum_{n=1}^{N}\phi_{n,s}B_{n,s}(\underline{r}) \tag{8}$$

where $\phi_{n,s}$ is the value of the level set in the *n*-th sub-domain at the *s*-th resolution level, $D_{n,s}$, $n=1,\ldots,N$, $s=1,\ldots,S$ [16]. As a consequence, the inversion procedure results as a multi-step process where the shape of the target is step-by-step improved by exploiting the information iteratively acquired about the *RoI*. The representative result of Fig. 3 shows the rough solution retrieved at the first step [Fig. 3(*a*)] and the final result at $s=2$ [Fig. 3(*b*)]. The scenario is characterized by the presence of two hollow cylinders, depicted by the red dashed lines, located in an investigation domain with side $L_D = 5\lambda$. The region of interest has been partitioned in $N = 841$ sub-domains and noisy data have been considered ($SNR = 20\,dB$, $M = V = 40$). For comparison purposes, Figure 3(*c*) shows the reconstruction obtained by the standard *LS*. As it can be observed, the shape is more accurately retrieved by means of *IMSA-LS* [Fig. 3(*b*) vs. Fig. 3(*c*)], which also obtains a misfit [Fig. 3(*d*)].

5. **Conclusions**

In the framework of microwave imaging, such a paper deals with the use of multi-resolution strategies to enhance the quantitative reconstruction in inverse scattering problems. The multi-resolution can be achieved both in quantitative and qualitative imaging problems by means of the exploitation of suitable basis functions or through multi-step procedures. In general, the multi-resolution approaches allows to reduce the number of unknowns in the problem at hand without limiting the achievable spatial resolution and the accuracy of the reconstructions. The approaches discussed in this paper proved to be effective when dealing with noiseless and noisy data, as well as in quantitative and qualitative imaging. They confirmed that a reduction of the number of unknowns is possible without degrading the quality of the final result.

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