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SIDELOBE REDUCTION THROUGH ELEMENT PHASE CONTROL IN SUB-ARRAYED ARRAY ANTENNAS

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Sidelobe Reduction through Element Phase Control in Uniform Sub-arrayed Array Antennas

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Abstract

When dealing with the synthesis sub-arrayed array antennas, the phase control, necessary for beam steering purposes, can be also exploited to reduce the undesired secondary lobes caused by the periodic spatial distribution of the amplitude excitations on the aperture when contiguous and identical sub-arrays are used. In order to determine the phase values of the array elements for a fixed sub-array amplitude weighting, the iterative projection method is adopted. Some representative results are shown to assess the effectiveness of the method.

Index Terms

Sub-arrayed array antennas, Iterative Projection Method, Phase control.

I. Introduction

The use of contiguous sub-arrays is a well-known technique to reduce the number of control elements in large array antennas when constrained feed networks are adopted [1]. Although homogeneous sub-arrays significantly simplify the antenna manufacturing with a reduction of the costs, the resulting array arrangement is characterized by undesired secondary lobes in the pattern region close to the main beam.

To avoid this drawback, different techniques aimed at breaking the periodicity of the aperture illumination have been proposed in the literature. For example, overlapped sub-arrays have been considered in [2] to generate low sidelobes. The same goal has been yielded in [3] by means of sub-array amplitude tapering, while the size of the sub-arrays has been optimized in [4]. On the other hand, the joint optimization of the sub-array sizes and weights has been considered in [5]. In regard to planar arrays, various methods based on the sub-array rotation [6], the use of a-periodically spaced sub-arrays [7], and a tiling strategy with sub-arrays of different shapes [8] have been presented.

In this letter, a technique based on the optimization of the phases of the array elements is considered to reduce the peak sidelobe level when amplitude weighting the sub-array ports. Since each element of the array has a phase shifter to electronically steer the beam pattern towards a desired direction, the same phase terms can be profitably used low sidelobes thus avoiding additional hardware and costs. The proposed approach uses the iterative projection method (IPM), also called intersection approach [9], to find the phase values through an alternate projection of the illumination function on the aperture to the far-field pattern and vice versa, until the distance between the actual pattern and the desired one does not exceed a numerical convergence threshold.

The paper is organized as follows. The synthesis problem is mathematically formulated in Sect. II where the procedure for the elements phase control is also described. The results from some representative experiments are reported and discussed in Sec. III to demonstrate the effectiveness of the proposed method. Eventually, some conclusions are drawn and future developments are envisaged (Sect. IV).

II. Mathematical Formulation

Let us consider a linear array of $N = S \times M_q$ equally-spaced elements placed along the z axis. The array elements are grouped into S uniform sub-arrays. Each sub-array has the same number of elements, M_q, and the q-th sub-array has an amplitude weight w_q, $q = 1, ..., S$. A phase shifter is located at the input port of each radiating element as shown in Fig. 1. The effective (complex) excitations are supposed to be symmetric with respect to the physical center of the antenna,

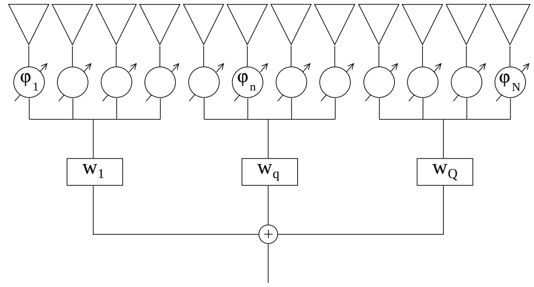

Fig. 1. *Array Architecture Geometry* - Example of a uniform sub-arrayed linear array antenna.

$\underline{A} = \left\{a_n = a_{-n};\, a_n = A_n e^{i\varphi_n};\, n = 1, ..., N\right\}$, being $N = Q \times M_q$ and $Q = \frac{S}{2}$. It is worth noticing that $A_n = w_q$ if the n-th element belongs to the q-th sub-array. Accordingly, the array factor is given by

$$AF\left(\theta\right) = 2\sum_{q=1}^{Q} w_q \sum_{m=(q-1)M_q+1}^{qM_q} e^{i\varphi_m} cos\left\{kz_m cos\left(\theta\right)\right\} \tag{1}$$

where $k = \frac{2\pi}{\lambda}$ is the free-space wave number, $z_m = d\left[\frac{2m-1}{2}\right]$ denotes the element location with respect to the physical center of the antenna, d is the inter-element spacing, and θ is the angular rotation with respect to the boresight direction.

For fixed sub-array amplitude weights (i.e., $w_q = \hat{w}_q$, $q = 1, ..., Q$), (1) can be optimized by properly modifying the phase values of the array elements φ_n, $n = 1, ..., N$. Because of the symmetry, only half of the elements are involved in the synthesis process. Towards this end, the IPM [9] is adopted. As a matter of fact, the IPM is an effective technique based on an iterative sequence of projections through Fourier transformations between the space of the array excitations $S^{\underline{A}}$ and that of far-field patterns S^{AF}. The space S^{AF} is defined as follows:

$$S^{AF} = \left\{AF\left(\theta\right):\; LM\left(\theta\right) \leq \left|AF\left(\theta\right)\right| \leq UM\left(\theta\right)\right\} \tag{2}$$

where $UM\left(\theta\right)$ and $LM\left(\theta\right)$ are an upper mask and a lower one, respectively, defined on the visible region of the antenna $\theta \in [-90^o; 90^o]$. As far as the space of the element excitations is concerned, the amplitude coefficients are constrained to the values of the sub-array weights $\hat{w}_q$. On the other hand, the element phases are unconstrained quantities belonging to the range $\varphi_n \in [-\pi; \pi]$, $n = 1, ..., N$. As for the phase control of sub-arrayed antennas, the IPM works according to the following procedure:

- **Step 0** - *Initialization*. At the first iteration ($i = 0$), the phase values of the array elements are set to $\varphi_n = 0$, $n = 1, ..., N$, while their amplitudes (A_n, $n = 1, ..., N$) are assumed to be equal to the corresponding sub-array amplitude weights, $A_m = \hat{w}_q$, $m \in [(q-1)M_q + 1;\, qM_q]$, $q = 1, ..., Q$;
- **Step 1** - *Pattern Generation*. The array factor $AF^{(i)}\left(\theta\right)$ is computed as the Fourier transform of the current set of excitations $\underline{A}^{(i)}$;
- **Step 2** - *Projection on to Patterns Space*. The solution $AF^{(i)}\left(\theta\right)$ is projected onto the closest point of S^{AF} by imposing $AF^{(i)}\left(\theta\right) = UM\left(\theta\right)$ if $AF^{(i)}\left(\theta\right) > UM\left(\theta\right)$ and $AF^{(i)}\left(\theta\right) = LM\left(\theta\right)$ if $AF^{(i)}\left(\theta\right) < LM\left(\theta\right)$ to obtain a projected solution $AF_{PR}^{(i)}\left(\theta\right)$ belonging to S^{AF};
- **Step 3** - *Fitness Evaluation*. The distance between $AF^{(i)}\left(\theta\right)$ and its projection $AF_{PR}^{(i)}\left(\theta\right)$ is computed

$$\Psi^{(i)} = \Psi\left(\varphi_n^{(i)}\right) = \frac{\left\|\left|AF^{(i)}\left(\theta\right)\right| - \left|AF_{PR}^{(i)}\left(\theta\right)\right|\right\|^2}{\left\|AF^{(i)}\left(\theta\right)\right\|^2}\;; \tag{3}$$

- **Step 4** - *Convergence Check*. The algorithm is stopped when either a maximum number of iterations I or the value of the cost function $\Psi^{(i)}$ is smaller than a user-defined threshold Ψ^{TH}. Accordingly, $\underline{A}^{opt} = \underline{A}^{(i)}$ and $AF^{opt}\left(\theta\right) = AF^{(i)}\left(\theta\right)$. Otherwise, go to Step 5;

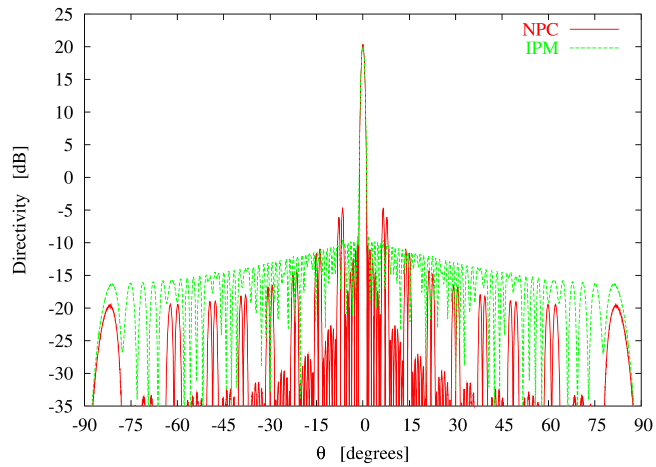

Fig. 2. *Experiment 1* ($N = 128$, $Q = 4$) - Beam patterns synthesized with (IPM) and without phase control (NPC).

TABLE I
Experiment 1 ($N = 128$, $Q = 4$) - PERFORMANCE INDEXES.

$Approach$	NPC	IPM
SLL $[dB]$	-25.1	-29.2
D_{max} $[dB]$	20.3	20.1
BW $[deg]$	1.01	1.05
η_T	0.843	0.785

- **Step 5** - *Projection on to Excitations Space*. The iteration index is updated, $i \leftarrow i + 1$, and a new set of excitations $\underline{A}^{(i)}$ is derived through the inverse Fourier transform of $AF_{PR}^{(i-1)}(\theta)$ while keeping constant the array amplitudes. Go to Step 1.

III. Numerical Results

To demonstrate the effectiveness of the exploitation of the phase control, some preliminary results concerned with the synthesis of pencil beams are shown and discussed, in a comparative fashion, with standard solutions. As an additional comment, it is worth noting that although the L2 pattern approximation of an optimal pencil beam can be computed using pure real excitations (under the considered symmetry constraints), complex excitations are looked for in this case since the approach is aimed at fitting user-defined fixed constraints.

The first experiment considers a uniform linear array having $N = 128$ elements equally spaced of $d = \lambda/2$. According to the sub-array strategy, the array is partitioned into $S = 8$ uniform and contiguous sub-arrays with a Taylor tapering ($SLL = -30\,dB$, $\overline{n} = 4$) [10] at the sub-array output ports. In such an example, the optimization of the element phases is aimed at synthesizing a beam pattern with SLL lower than that without phase control (called *static mode*). To this purpose, the masks $UM(\theta)$ and $LM(\theta)$ were set to obtain a pattern with exponentially decreasing sidelobes and maximum SLL equal to $-30\,dB$, while maintaining the same beamwidth $BW = 1$ $[deg]$ of the amplitude-only sub-array tapering. The IPM was run for a maximum of $I = 5000$ iterations and the threshold on the cost function has been fixed to $\Psi^{TH} = 10^{-10}$.

The optimized solution obtained after $t = 212.1$ $[sec]$ on a $1.7\,GHz$ PC with MB of RAM has a cost function value equal to $\Psi^{opt} = 1.36 \times 10^{-8}$. For comparison, the directivity patterns of the solutions synthesized with (IPM) and without phase control (NPC) are shown in Fig. 2. From the analysis of the values of the pattern indexes in Tab. I, it is worth pointing out that the highest secondary lobes close to the main lobe are reduced by more than $4\,dB$ ($SLL^{NPC} = -25.1\,dB$ vs. $SLL^{IPM} = -29.2\,dB$), while maintaining a close and high value of directivity along the boresight direction ($D_{max}^{NPC} = 20.3\,dB$ and $D_{max}^{IPM} = 20.1\,dB$). As far as the array efficiency is concerned [1], it turns out that $\eta_T^{IPM} = 0.785$ and $\eta_T^{NPC} = 0.843$.

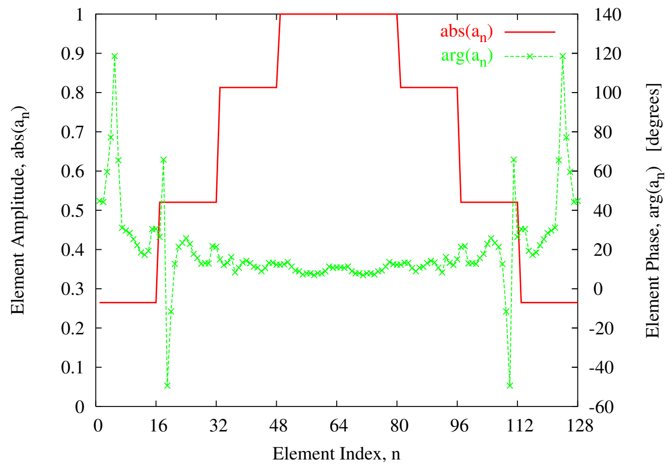

Fig. 3. *Experiment 1* ($N = 128$, $Q = 4$) - Plots of the amplitude and phase values of the element excitations.

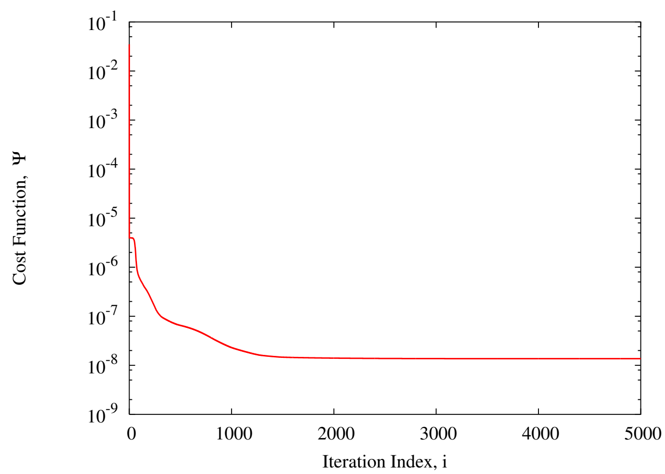

Fig. 4. *Experiment 1* ($N = 128$, $Q = 4$) - Behavior of the cost function through the optimization process performed by means of the IPM.

Figure 3 gives a representation of the synthesized element excitations. For the sake of completeness, the plot of the cost function during the optimization process is shown in Fig. 4. As it can be observed, the cost function value only marginally decreases after $i = 2000$. Therefore, the use of a termination criterion, instead of that described at Step 4, based on the stationariness of the cost function could further improve the efficiency of the IPM.

The second experiment is concerned with the optimization of a smaller array and is aimed at evaluating the impact of having a reduced number of degree of freedom (i.e., control elements φ_n, $n = 1, ..., N$) in the synthesis problem at hand. Towards this purpose, a uniform array of $N = 32$ elements and inter-element spacing equal to $d = \lambda/2$ is taken into account. Eight elements are assigned to each of the $S = 4$ identical sub-arrays, which characterize the constrained feed network at hand. Likewise the previous test case, the sub-array weights w_q, $q = 1, ..., S$, have been computed by sampling of the Taylor distribution with $SLL = -30\,dB$ and $\overline{n} = 2$ [10] such that $w_1 = w_4$ and $w_2 = w_3$ in order to deal with an array made of two identical halves.

As far as the IPM is concerned, the time required to get the final solution has been equal to $t = 151.3$ $[sec]$. The optimized values of the array excitations are shown in Fig. 5. Such a solution has a fitness value equal to $\Psi^{opt} = 3.48 \times 10^{-10}$ and the corresponding pattern is shown in Fig. 6. For comparison purposes, the pattern generated without phase control and only with real excitations is shown, as well. As expected, the enhancement in terms of SLL reduction is lower than that gained in the previous experiment because of the smaller number of control elements (i.e., degrees of freedom). Table II shows the IPM pattern has a $3\,dB$ reduction in sidelobe level countered by a $0.5\,dB$ reduction in directivity. Despite the small number of sub-arrays ($Q = 2$), these results further confirm the effectiveness of the proposed method in designing array antennas with

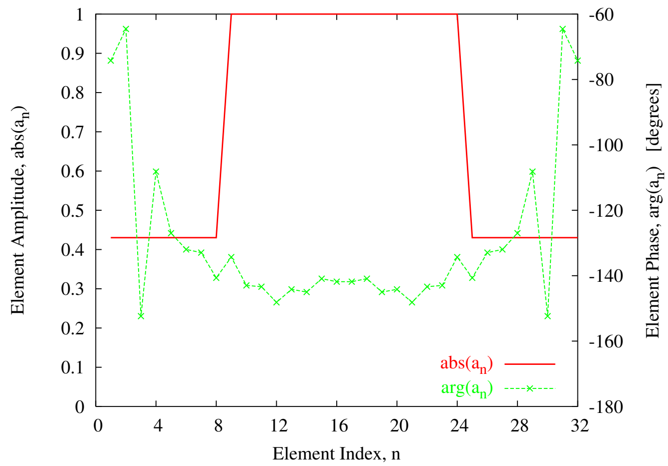

Fig. 5. *Experiment 2* ($N = 32$, $Q = 2$) - Plots of the amplitude and phase values of the element excitations.

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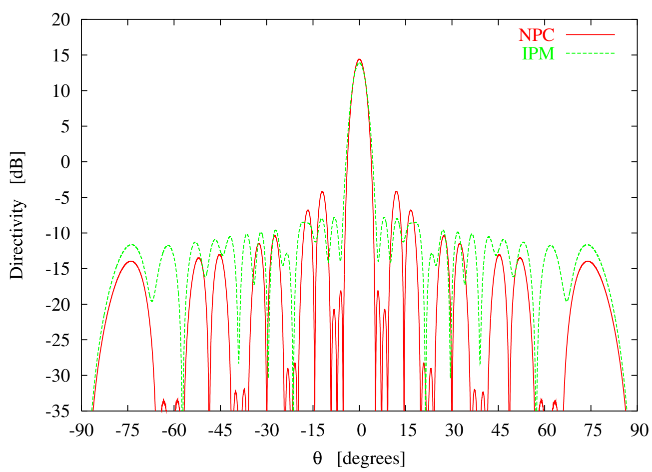

Fig. 6. *Experiment 2* ($N = 32$, $Q = 2$) - Beam patterns synthesized with (IPM) and without phase control (NPC).

simple feed network architectures and enhanced performances in terms of SLL reduction.

Finally, because of the reduced number of elements, let us also compute the roots ψ_k, $k = 1, ..., N-1$, of the polynomials characterizing the array factors of the two patterns of Fig. 6. The real $Re(\psi_k)$ and imaginary $Im(\psi_k)$ parts of these roots are shown in Fig. 7. It is interesting to observe that some NPC roots are shifted outside the unit circle to synthesize an IPM pattern with lower secondary lobes. The plot of the IPM pattern (Fig. 6 - dashed line) has a reduced number of nulls compared to those of the NPC curve. Consequently, it turns out that, as a side effect, it would be more difficult to locate a suitable null along an arbitrary angular direction, while maintaining a low SLL, by using a phase-only control strategy [11].

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IV. Conclusions

In this letter, the element phases are optimized to reduce the maximum sidelobe level of linear arrays with amplitude tapers at the sub-array ports. In order to break the periodic behavior of the array excitations, the control of the phases of the array elements has been profitably exploited taking into account that phase shifters are usually present in the feed network to steer the beam pattern. As a matter of fact, controlling the phases of the array elements provides additional degrees of freedom for the array synthesis problem for optimizing either the sub-array weights and/or the antenna geometry. Preliminary results have been reported to show the efficiency of the proposed approach.

TABLE II
Experiment 2 ($N = 32$, $Q = 2$) - PERFORMANCE INDEXES.

$Approach$	NPC	IPM
$SLL\ [dB]$	-18.6	-21.7
$D_{max}\ [dB]$	14.4	13.9
$BW\ [deg]$	3.85	4.20
η_T	0.863	0.766

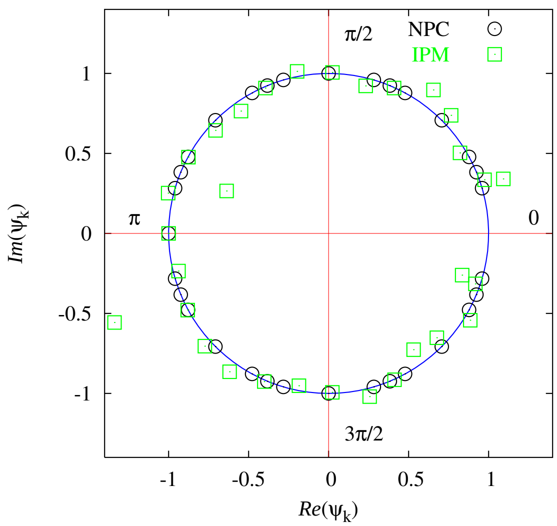

Fig. 7. *Experiment 2* ($N = 32$, $Q = 2$) - Real $[Re\,(\psi_k)]$ and imaginary $[Im\,(\psi_k)]$ parts of the roots of the polynomial of the array factor.

Further researches will be devoted to extend the proposed approach to the synthesis of shaped beam arrays as well as to assess its feasibility and efficiency in radar applications.

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