GROWTH, REAL INTEREST, EMPLOYMENT AND WAGE DETERMINATION

Luigi Bonatti

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dott. Paolo Maggioni
Dipartimento di Economia
Università degli Studi
Via Inama 5
38100 TRENTO ITALY
ABSTRACT: Structurally similar economies sharing the same wage-setting institutions tend to grow at the same rate, thus preserving their differences in levels of output per capita and employment rate, if the value of the workers’ outside options is rigid and equal across economies. In this case, i) multiple balanced growth paths can be possible, ii) the sustainable rate of growth is higher in economies with competitive wage determination than it is in unionized economies, and iii) this growth differential becomes larger in the presence of an integrated capital market. As the workers’ outside option depends on fiscal transfers responding to changes in levels of output and employment, there is convergence in levels across structurally and institutionally similar economies. In this case, i) economies with competitive wage setting converge at higher levels of output per capita and employment rate than do unionized economies, ii) economies with the same wage-setting institutions converge at higher levels if the tax rate and the fiscal transfers in favour of the jobless are reduced, and iii) the steady-state (“natural”) rate of employment characterizing a regional economy is higher—other things being equal—the lower is the steady-state rate of employment of the other region, when a central government pays equal benefits to all jobless households living in this two-region economy.

KEY WORDS: Convergence, on-the-job training, physical capital, human capital, skilled labor, multiple equilibria, fiscal transfers, coordination failure.

JEL CLASSIFICATION NUMBERS: J24, J51, J64, O41, R11.

*Università di Trento and Italian Academy for Advanced Studies in America at Columbia University. E_mail: LBONATTI@GELSO.UNITN.IT
INTRODUCTION

What growth and employment paths are sustainable depend on the structural and institutional characteristics of the economy. In particular, the modalities of wage determination can affect the process by which resources strategic for growth i.e., physical capital and skilled labor, are accumulated, thus influencing growth and employment patterns.

Dealing with these issues, the general-equilibrium model presented in this paper may help explaining the different growth and employment performances of the advanced economies in recent years. Its main aims are the following:

i) To show how initial differentials in output per capita and employment rate between structurally similar economies sharing the same labor-market institutions cannot vanish in the long run if the opportunities of workers without jobs are rigid and equal across these economies (typically because home activities have the same value for a worker no matter where s/he is located). In contrast, when the workers’ outside opportunities depend on fiscal transfers responding to changes in the output and employment performances of each economy, initial differences between economies that are structurally and institutionally similar tend to disappear in the long-run, with the economies converging to the same steady-state level of output per capita and to the same “natural” employment rate.

ii) To shows that with the workers’ outside opportunities equalized across economies, an economy characterized by competitive wage determination tends to grow at a

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higher rate than an economy in which unions negotiate wages. Furthermore, it shows that when the workers’ outside opportunities depend on fiscal transfers, the steady-state level of output per capita and the “natural” employment rate are higher if the wages are determined competitively rather than being union determined or—given the modality of wage determination—if the fiscal transfers are lower. Finally, in the context of a two-region economy with a central government paying equal benefits to all jobless households, it shows that the natural rate of employment characterizing a region is higher—other things being equal—the lower is the natural rate of employment of the other region.

iii) To show that the existence of an integrated capital market favours the economy enjoying a competitive advantage because of its labor-market institutions, lowering the steady-state rate of growth of the economy whose modalities of wage determination are less favourable to growth.

iv) Again in a context where the value of the workers’ outside opportunities does not respond to the levels of output and employment, to show that multiple balanced growth paths are possible, so that more than one long-term interest rate is consistent with market “fundamentals”.

The paper is organized as it follows: Section 1 provides the factual and theoretical premises of the paper. Section 2 presents the basic model. Section 3 characterizes an equilibrium path and solves the model for the case in which the insiders’ unions have control over the wage setting process. Section 4 solves the model for the case in which there are market-determined wages. Section 5 characterizes the equilibrium path as a country with union-determined wages and a country with market-determined wages coexisting in the presence of perfect capital mobility. In section 6 the possibility of multiple equilibrium paths is discussed, while
in section 7 the assumption is relaxed that the workers’ outside opportunities do not respond to the conditions prevailing in the economy. The final section summarizes the main results of the paper.

1. BACKGROUND

In the last two decades, differences in GDP per capita between European regions have appeared to be quite persistent (Neven and Gouyette 1995, Fagerberg and Verspagen 1996). Typically, regions with lower level of GDP per capita tend to be those having the higher rate of unemployment (Fagerberg, Verspagen and Caniëls 1997). Moreover, it is often the case that depressed areas exhibit lower rates of labor force participation\(^1\) and have relatively large underground economies.\(^3\) However, in spite of long-lasting (and rising) differentials in regional unemployment rates, interregional migration flows have declined and then remained very low in the last two decades.\(^4\) In Southern Europe, in particular, where some of the regions with the highest unemployment rates are located, unemployment is closely concentrated among

\(^1\) In the case of Italy, there are many studies showing the lack of convergence in GDP per capita between southern regions and the rest of Italy in the last 25 years (GDP per capita in the South as a fraction of GDP per capita in the rest of Italy was .607 in 1972 and .567 in 1997). Among these studies, see Cosci and Mattesini 1995, Boltho et al. 1995, Cellini and Scorcu 1997, Fabiani and Pellegrini 1997, Paci and Saba 1998.

\(^2\) This is particularly true for Italy, where there is a significant and persistent differential in participation rates between the South and the rest of the country, which is mainly due to the difference in the female participation rate (in 1995 only 33.7 % of the southern women aged 15-64 were in the labor force, while for Italy as a whole the corresponding figure was 43.2%). As it is well known, changes in labor force participation seem to play a larger role in Europe than in the USA to adjust movements in labor demand (see Decressin and Fatás 1995).

\(^3\) In 1993 about one third of total labor units in South Italy were estimated to be irregular (against 18% in the rest of the country).

\(^4\) This is true both for Italy (Attanasio and Padoa Schioppa 1991, Faini et al. 1996) and for Spain (Bentolila and Jimeno 1998).
young people and—especially in South Italy—among first-job seekers. Finally, there is a correlation between regional net fiscal transfers and regional unemployment (Obstfeld and Peri 1998), which may support the hypothesis that a relatively high government transfers in favour of households located in depressed areas have contributed to raise and keep high the reservation wage of the workers living in these areas, also depressing their participation rate and their propensity to migrate.

The existence of these fiscal transfers may be one of the reasons why wages are not particularly sensitive to local labor market imbalances. Moreover, in the presence of these massive transfers, it is doubtful that decentralizing the wage-bargaining process at the regional level will be sufficient to differentiate wages according to local labor-market conditions, since fiscal transfers and welfare entitlements tend to equalize the workers’ outside opportunities across regions of the same country.

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5 At the end of 1997, 44.7% of the Italian unemployed were first-job seekers, and 61% were aged less than 30.


7 In Italy, regional differentials in unemployment rates do not appear to exert a significant influence on wages (see Faini 1995). In the formal segment of the economy, where minimum wages are negotiated at national level, some evidence shows that wage determination is not affected by the southern unemployment rate (Bodo and Sestito 1991, Brunello et al. 1999). However, even in the small-firm sector, wages are not very sensitive to excess supply in the local labor market (Casavola et al. 1995). Also in Spain there is evidence showing that in spite of the large and persistent differentials in regional unemployment rates, real wages do not differ significantly between high-unemployment and low-unemployment regions (Mauro et al. 1999).

8 For a model showing that centralized wage-setting should be discouraged and skilled labor mobility should be favoured in order to foster regional convergence, see Faini 1999.
However, it is likely that generous fiscal transfers are not the only explanation of the relatively high reservation wages that are observed in some depressed areas. In these areas, indeed, there are fewer opportunities for using the skills acquired while working in entry-level jobs. This lowers the expected return on the investment that a worker undertakes when s/he accepts a relatively low entry-wage in order to receive training on the job, thereby raising the minimum wage at which s/he is willing to accept such a job. In its turn, this dissuades firms from locating their plants in these areas. Thus created is a vicious circle comprising low probability of finding “good” jobs, low investment for the creation of new good jobs and for active participation in the good labor market, coupled with a relatively extended informal economy offering badly-paid work opportunities for unskilled workers and little possibility for the acquisition of skills.

The existence of such “traps” is explained in the literature by stressing the strategic complementarities between investment in physical capital, in R&D, or in the creation of jobs, on the one hand, and investment to acquire the required human capital and to conduct a job search on the other. These strategic complementarities depend on the fact that one type of investment would have higher expected returns and would have been increased if also the other type of investment were increased (e.g. see Cooper and John 1988). Hence, the presence of these complementarities may generate multiple equilibria and lead to coordination failures: in the absence of some institutional device coordinating the individual expectations and actions, decentralized

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9 In other words, the small number of good firms restricts the availability of career paths based on job-to-job mobility, which would allow workers to reap the returns on the investment made by accepting a lower entry wage.

10 The discouraging effect exerted by the low probability of finding a job on female labor market participation is documented by Casavola et al. 1995.
decision makers can give rise to Pareto-suboptimal outcomes (Burdett and Smith 1995, Acemoglu 1996, Redding 1996, Snower 1996).\textsuperscript{11}

Given the possibility of multiple equilibria, the present paper invokes the centrality of the capital market in selecting the trajectory of the employment level. It accepts the idea that in contemporary capitalism the conventional opinions held by capital-market participants may have a role in shaping the future of an economy.\textsuperscript{12} Since more than one equilibrium paths is consistent with market fundamentals and institutions, the economy will move along an equilibrium path characterized by high growth only if market participants are convinced that this growth scenario is sustainable—that is, if they believe that firms' future profitability is able to guarantee a sufficiently high rate of return on the larger amount of resources invested. In these circumstances, the upward pressure on the cost of capital exerted by increased demand for capital is accomodated by an adequate increase in capital supply. The externalities positively correlated to the growth rate of the economy are crucial in preventing the rate of return on capital from falling along a path characterized by more capital accumulation, thus validating the optimistic opinions prevailing in the capital market. In other words, it is only if the capital-market participants believe in the sustainability of a high growth rate that they will generate a flow of resources sufficient to finance (at an acceptable cost for the productive sector) buoyant investment activity by firms.

\textsuperscript{11} In Saint-Paul (1994a and 1994b), the workers may prefer the low-skill equilibrium to the high-skill equilibrium, because the higher demand for education reduces national savings, pushing up real interest and unemployment rates.

\textsuperscript{12} Ciocca and Nardozzi (1995) convincingly discuss the conventions underlying the attitudes of the financial markets in order to explain the movements of real interest in the OECD countries since the early 1960s.
This paper therefore challenges the proposition that the interest rates of efficient capital markets are always able to coordinate intertemporal activities appropriately (see Leijonhufvud 1995). Since the economy can gravitate around multiple equilibrium paths, appropriate policies and institutions are advisable because of their role in averting a coordination failure and in achieving a Pareto-superior outcome (Hahn and Solow 1995, chapt.7; Colander and van Ees 1996). Indeed, as more than one equilibrium path is possible, it is not necessarily the case that a long-term trade-off between employment and real wage emerges: along a trajectory characterized by a higher rate of growth, both the employment level and the average real wage tend to be higher than when the economy follows a low-growth path. In accordance with this approach, the role of economic policy is to provide a consistent and reliable framework able to enforce the convention favourable to a high growth scenario, convincing the financial markets of its sustainability (see Ciocca and Nardozzi 1995).

It is hard to believe that structurally similar economies may exhibit non-vanishing differentials in per capita product for long periods of time only because of the self-fulfilling nature of rational expectations. An explanation stressing the crucial role of the “animal spirits” as determinants of growth should consistently argue that it would be sufficient to "help" the agents to coordinate their expectations—thereby enabling them to converge on the desired growth path—for the undesired disparities among economies to be rapidly eliminated. Given that these disparities may persist for decades, it is unlikely that such jumps can be made possible simply by helping the agents to form the "right" expectations. In other words, the existence of persistent differentials across regions or countries prompts one to search for plausible mechanisms able to amplify the effects of transitory shocks and to determine
cumulative processes. These mechanisms rely on the fact that the endowments of resources strategic for growth depend on history: the body of knowledge, the state of technology and the stock of human, social and physical capital result from rates of innovation, discovery and accumulation in the past. Therefore, persistence can be generated because a period of high (low) growth has left the economy with increased (depleted) stocks of the resources required for a self-propelled growth process, with long-lasting effects on the economy.

The persistence-causing mechanism formalized in this paper emphasizes that the availability of experienced and trained workers is not independent of history, because experience and training can be acquired by working. Therefore, the existence in the past of firms which offered opportunities for training on the job, and for acquiring experience and abilities, increases the current endowment of human capital in the economy, thereby boosting productivity growth. This implies that formal education cannot substitute for learning by doing, even if it is a pre-condition for it:

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13 Empirical data seem to confirm the contribution made to total factor productivity by the learning process which takes place when machinery and technologies are used (see, for example, De Long and Summers 1992). There is also empirical support for the hypothesis that a shortage of qualified workers has negative effects on productivity growth (for microeconometric evidence concerning the United Kingdom, see Haskel and Martin 1996).

14 The informal acquisition of skills and the availability of these skills in certain areas because of previous settlements, which sometimes originated in pre-industrial traditions, have been very important for the formation and evolution of Italian industrial districts, that consist mainly of networks of small firms. These districts, which are located in the northern and central regions of the country (with very few exceptions), constitute the core of the Italian manufacturing sector and generate a very large portion of Italian exports. Conversely, the depressed South of Italy is well endowed with formally educated people (in the South the share of population with higher school degrees is even higher than in the richer North!). This notwithstanding, entrepreneurs often complain about the difficulty of finding in the South the skilled workers that they need in the South (Banca d’Italia 1995, Istituto G. Tagliacarne 1996). Confirming the existence of a “low-skill, bad-job” trap, one should note that—-in an environment offering relatively few good-
possession of basic formal education is necessary to be able to learn on the job. In the model, this strict complementarity is captured by assuming that a worker can be hired by a good firm giving him/her training on the job only if s/he invested to become “trainable”, i.e., if s/he invested to participate actively in the good job market.

The model here presented allows to derive permanent differentials in the steady-state rate of growth of output and employment or in the steady-state levels of output and employment across economies with different structural parameters and wage-setting processes. Therefore, it is consistent with both a “full” hysteresis approach and an intertemporal equilibrium endogenous natural rate approach to unemployment. We have mentioned that persistent differentials in output per head and employment rate between areas structurally and institutionally similar but endowed with different initial stocks of human and physical capital can become permanent only if the long-term adjustment process based on the relative weakening of the market position of workers located in the poorer areas cannot take place. In this case, the paper points to the conclusion that in the long-run the growth rate of employment is higher when wage determination responds, not to insiders’ pressure, but to external market forces. Moreover, an area in which wages are competitively determined is better able to exploit its competitive advantage in the presence of an integrated capital market because it is able to attract capital from the area in which insiders’ pressure

job opportunities in the private sector--a disproportionately large number of southern Italians obtain school degrees that can give access almost exclusively to public jobs, thus increasing the pressure to inflate the public sector (Paganetto and Scandizzo 1996, Bonatti and Borzaga 1998). This complementarity is supported by OECD 1991, which emphasizes that on average less formal schooling seems to lead to more limited training opportunities and possibilities to augment human capital.
is stronger, thus damping the steady-state rate of growth of the less competitive area. However, even in the case where a long-term adjustment can take place, and the differences in output per head and employment rate between areas structurally and institutionally similar tend to disappear, the paper suggests that in the long run the level of output per capita and rate of employment is higher when wages are competitively determined rather than subject to insiders’ pressure or—given the modality of wage determination—when fiscal transfers in favour of the jobless are reduced. In this respect, this paper seeks to give rigour to the widespread notion that labor-market institutions more responsive to competitive forces are conducive to better growth and employment performances, and thereby improving our understanding of why the U.S. experience has differed quite dramatically from that of continental Europe in recent years.

16. In recent years, the comparative advantage enjoyed by the USA versus continental Europe in terms of labor market institutions has been often mentioned as one of the feature explaining the disappointing performance of many European countries in attracting direct investment vis-à-vis the ability of the U.S. to attract it.

17. The long-term adjustment mechanism stressed here follows Phelps (1994) in emphasizing the importance for workers of relying on sources of support that do not depend on having a regular job. In Phelps, convergence in the natural rate of unemployment between structurally and institutionally similar economies can occur because the real and financial assets (together with the welfare entitlements) held by the workers located in an economy affected by a high unemployment rate tend to decline relatively to the non-labor sources of support available to workers located in a similar economy characterized by lower unemployment.

18. Among the recent papers focusing on the role of labor-market institutions in explaining cross-country differences in unemployment rates, see Nickell 1997, and Blanchard and Wolfers 1999. The model here presented seeks to take into account of the four labor-market features that according to Nickell (1997) are associated with high unemployment in Europe: i) generous transfers in favour of the jobless, ii) high unionization with wages bargained collectively and without coordination, iii) high overall taxes, and iv) poor educational standards at the bottom end of the labor market.
2. THE BASIC MODEL

We consider an infinite-horizon economy in discrete time. The relevant agents are the firms, the investors and the workers.

Population’s dynamics

Individuals are finitely lived: they have a strictly positive and constant probability $\sigma$, $0<\sigma<1$, of dying in each period $t$. Thus, the probability of dying in a certain period is assumed to be independent of the age of the individual; and it is also assumed that the mortality rate of each large group of individuals does not fluctuate stochastically even though each individual’s lifespan is uncertain. This implies that at the end of $t$ a constant fraction $\sigma$ of individuals belonging to each group and living in location $i$, $i\in[0,n]$, $n>0$, dies, while a new cohort is born at the beginning of the following period.

The firms

There is a continuum--of measure $n$--of locations. In each location $i$ there is a large number (normalized to be one) of identical firms. Locations differ with respect to the specific shock affecting them in each period. Indeed, in each period $t$ the representative firm located in $i$ (the "i firm") produces the output $Y_{it}$ according to the constant-returns-to-scale technology

$$Y_{it} = x_{it} K_{it}^{1-\alpha} (S_{it} + \Omega A_{it})^\alpha, \quad 0<\alpha<1, \quad 0<\Omega<1,$$

where $x_{it}$ is a random variable taking a value in period $t$ which is specific to the $i$ location, $K_{it}$ is the physical capital that the $i$ firm borrowed at the end of the previous period to carry out production in $t$, $S_{it}$ are the experienced workers (the "skilled workers") employed by the $i$ firm in $t$, $A_{it}$ are the newly hired workers (the
"apprentices") of the i firm in t. Note that the apprentices are less productive than the experienced workers ($\Omega < 1$), and that aggregate output is given by $Y_t = \int_0^n Y_i dt$.

The random variable $x_{it}$ is assumed to be uniformly distributed on the interval [0,n]. Moreover, it is identically distributed across locations and periods, and independently distributed across periods. In each t, $x_{it}$ takes a different value in each location, with $x_{it}$ varying continuously across locations. This implies that the average value of $x_{it}$ across locations is not a random variable and does not fluctuate in time, even though individual firms are uncertain about their local $x_{it}$ (no aggregate uncertainty).\footnote{In other words, if $K_i = \bar{K}/n$, $S_i = \bar{S}/n$ and $A_i = \bar{A}/n$, $\forall i$ and $\forall t$, then $Y_t = \frac{\alpha}{2} \bar{K}^{1-\alpha} (\bar{S} + \Omega \bar{A})^{\alpha}$, $\forall t$.}

The period net profits $\pi_{it}^n$ (net of the cost of capital) of the i firm are given by:

$$\pi_{it}^n = \pi_{it}^g - [(1+r_t)-(1-\delta)]K_{it} = Y_{it}-v_{it}S_{it}-e_{it}A_{it} - [(1+r_t)-(1-\delta)]K_{it}, 0<\delta<1, \quad (2)$$

where $\pi_{it}^g$ are the firm's gross profits, $v_{it}$ is the real wage paid by the i firm to the skilled workers employed in t, $e_{it}$ is the entry wage paid by the i firm to the apprentices hired in period t, $\delta$ is a capital depreciation parameter, and $r_t$ is the (real) interest rate, i.e. the market rate at which firms borrowed capital at the end of the previous period. Interest payment and reimbursement of principal are due at the end of t. The interest rate is unique because capital is perfectly mobile across locations at the end of each period, while mobility is infinitely costly within the period: once
borrowed and installed at the end of t-1, a firm’s capital stock must remain fixed until the end of t.

The investors

There is a large number (normalized to be one) of identical investors who are the firms’ owners: for simplicity and without loss of generality, we assume that all investors are entitled to receive an equal share of the firms’ net profits. Moreover, the investors are the owners of the firms’ productive assets: investors must decide in each t what fraction of their gross returns on wealth to spend on consumption rather than on buying productive assets to be lent at the end of the period to firms. Hence, the problem of the representative investor amounts to deciding a contingency plan for consumption $C_t^{\text{in}}$ and holding of productive assets $K_{t+1}$ in order to maximize the lifetime expected sequence of discounted period utilities $g(C_t^{\text{in}})$:

$$E_0 \left( \sum_{t=0}^{\infty} \theta^t g(C_t^{\text{in}}) \right), \theta = \tau(1 - \sigma), 0 < \tau \leq 1, g' > 0, g'' \leq 0,$$

subject to $K_{t+1} + C_t^{\text{in}} \leq (1 + r_t)K_t + \pi_t^n$, $K_t = \int K_{it}(di)$, $\pi_t^n = \int \pi_{it}^n(di)$, $K_{i0}$ given $\forall i$.

In (3), $\tau$ is a time-preference parameter, $K_t$ and $\pi_t^n$ are, respectively, aggregate capital and aggregate (net) profits. Furthermore, $E_0$ is an expectation operator conditional on the information available in t as the values taken by $x_{it}$ across locations are known. Expectations are rational, in the sense that they are consistent with the model and are generated by optimally processing the available information. Finally, for simplicity and without loss of generality, we rule out the existence of actuarially fair annuities paid to the living investors by a financial institution collecting their wealth as they die: the wealth of someone who dies is inherited by some newly born individual (accidental bequests).
Skilled workers are those who have been trained on the job while working in a firm for at least one period. In contrast, apprentices are workers with no work experience, but who have been hired by a firm after having invested to participate actively in the labor market. In their working lives, workers never lose the general skills that they have acquired. Being general, the skills acquired on the job are perfectly transferable. Thus, the evolution of the skilled labor force in the entire economy is governed by
\[
M_{t+1} = (1-\sigma)(M_t+A_t), \quad M_t = \int_0^n M_i^t di, \quad A_t = \int_0^n A_i^t di, \quad M_0 \text{ given},
\]
where \(M_i^t\) are the skilled workers located in \(i\) at the beginning of period \(t\).

As in Blanchflower and Oswald (1994), workers choose location ex ante (at the end of \(t-1\)), while firms decide on labor input once uncertainty is resolved. As for capital, labor is perfectly mobile across locations at the end of each period, while mobility is infinitely costly within one period. This short-term immobility implies that those workers not employed in their location are not employed at all.

At the beginning of period \(t\), a skilled worker located in \(i\) has the following period expected utility:
\[
u_{it}^{sk} = E_i[p_{it}u(v_{it}) + (1 - p_{it})u(w_t)], \quad u' > 0, \quad u'' \leq 0, \quad v_{it} \geq w_t,
\]
where \(E_i\) is an expectation operator conditional on the information available at the beginning of period \(t\) (as the realization of \(x_{it}\) is not yet known), \(w_t\) is the monetized value of the workers’ outside option depending only on the value of non-market (home) activities (this assumption will be removed in section 7), and \(p_{it}\) is the fraction of the skilled workforce located in \(i\) that is employed in period \(t\):
\[
p_{it} = \begin{cases} 
\frac{S_{it}}{M_{it}} & \text{if } S_{it} \leq M_{it} \\
M_{it} & \text{otherwise.}
\end{cases}
\]

Note that \(w_t\) is equal across locations because it does not depend on local market conditions.

At the end of each period, a skilled worker may move to another location at no cost. Obviously, s/he locates where s/he can expect to enjoy the highest lifetime utility. Therefore, the discounted sequence of utilities that an optimizing skilled worker still alive at the end of \(t\) and located in \(i\) can expect to gain in the rest of his/her lifetime is given by

\[
U_{it}^{sk} = \beta E_t [u_{it}^{sk} + (1 - \sigma)U_{it+1}^{sk}], \quad 0 < \beta < 1. \tag{7}
\]

In (7), \(\beta\) is a time-preference parameter, and \(i^*\) is a location where a skilled worker can have the best lifetime prospects (a "best location"):\(^{20}\)

\[
U_{i^*t}^{sk} \geq U_{it}^{sk}, \quad \forall i. \tag{8}
\]

The entire income earned by the skilled workers working in the formal economy is devoted to consuming \(Y_t\):

\[
C_{it}^{sk} = S_{it}v_{it}, \tag{9}
\]

where \(C_{it}^{sk}\) is total consumption of the good \(Y_t\) by the skilled workers of location \(i\) in period \(t\).

**The unskilled workers**

At the beginning of each period, an unskilled worker located in \(i\) must decide whether to incur the utility loss associated with participation in the labor market (i.e., with searching for a job as an apprentice) or to remain out of the labor market: an unskilled worker can be hired by a firm only if s/he invests in labor market

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\(^{20}\) More than one location can share this status of best location. Obviously, a worker located in \(i^*\) will not move.
participation, i.e., only if s/he becomes “trainable”. An investment in search activity in period $t$ yields a strictly positive probability of being employed only in that period: the search must be repeated in subsequent periods if it is unsuccessful in the current period and the unskilled worker still wants to have some chance of getting a job. Hence, the investment made in order to participate in the labor market will be lost, if within one period, the worker does not find an entry job paid at least as his/her reservation wage: after having invested in search activity, a trainable worker will accept any job offer paying an entry wage larger than his/her reservation wage $e_{it}^{\min}$.

An unskilled worker who has decided not to search has the same lifetime prospects as a worker who is still unemployed after having incurred the utility loss entailed by this search. Therefore, an optimizing unskilled worker living in $i$ can expect at the beginning of $t$ to get the lifetime discounted sequence of utilities associated with the best available alternative:

$$U_{it}^{un} = \max \{ E_{it}[-h(c) + q_{it}(u(e_{it}) + (1 - \sigma)U_{it}^{sk}) + (1 - q_{it})(u(w_{it}) + (1 - \sigma)U_{it}^{un})],$$

$$E_{it}[u(w_{it}) + (1 - \sigma)U_{it}^{un}] \}$$

$h > 0, u(e_{it}) \geq u(e_{it}^{\min}) = u(w_{it}) - (1 - \sigma)(U_{it}^{sk} - U_{it}^{un})$. (10)

In (10), $-h(c)$ captures the disutility of search ($c$ is the monetized value of this disutility) and $q_{it}$ is the fraction of the trainable workforce located in $i$ which is hired in period $t$:

$$q_{it} = \begin{cases} 
\frac{A_{it}}{L_{it}} & \text{if } A_{it} \leq L_{it} \\
1 & \text{otherwise},
\end{cases}$$

(11)

where $L_{it}$ the trainable workforce located in $i$.

Note in (10) that $e_{it}^{\min}$ (the minimum wage at which a trainable worker is willing to accept a job offer) reflects also the expected increase in lifetime well-being conditional on receiving some training on the job and becoming a skilled worker. Indeed, $U_{it}^{un}$ is the discounted sequence of utilities that an optimizing unskilled
worker still alive at the end of \( t \) and located in \( i \) can expect to get in the rest of his/her lifetime:

\[
U_{it}^{un} = \beta E_t (\tilde{U}_{i^*t+1}^{un}),
\]

where a best location \( i^* \) for \( un \) unskilled worker must be such that

\[
U_{i^*t}^{un} \geq U_{it}^{un}, \ \forall i.
\]

The entire income earned by the apprentices is devoted to consuming \( Y_t \):

\[
C_{it}^{un} = A_{it} e_{it},
\]

where \( C_{it}^{un} \) is total consumption of the good \( Y_t \) by the apprentices of location \( i \) in \( t \).

**Union-determined wages**

We have two alternative institutional setups for wage determination. In the first one, an insider-outsider scenario is considered. In each location, the wages are determined by negotiations held at the beginning of every period between a local union unconcerned about the interests of workers with no work experience and the local employers' association. In this context it is immaterial whether the union is only concerned about the workers employed in the previous period, or about both the latter and those experienced workers who were laid off in previous periods. In fact, even if the wage setters do not care about the interests of the skilled workers on layoff, the latter put pressure on them, insofar as they are perfect substitutes and thereby reduce the job security of the employed.

The union operating in \( i \) negotiates the real wage that all the firms of \( i \) must pay to the experienced workers in employment, while each individual firm takes its decisions on the demand for labor and capital in full autonomy. This negotiation also concerns the entry wage, which is established as the fixed fraction \( \mu \) of the skilled workers' wage that firms must pay to the apprentices \( (e_{it} = \mu v_{it}) \). It is realistic to assume that the union does not allow the wage differential between skilled workers and apprentices fully to offset their productivity differential \( (\Omega < \mu \leq 1) \), so that any
incentive for the employers to replace experienced workers with apprentices is suppressed.\footnote{Burdett and Smith (1995) emphasize that the key assumption for the existence of a low skill trap is that an employer's profit flow is greater when employing a skilled worker than when employing an unskilled worker. Indeed, the fact that firms lay off unskilled workers before skilled workers is difficult to reconcile with the contention that unskilled workers are more profitable.}

The bargaining process can be represented as if each union unilaterally sets the real wage in the awareness of its impact on the local firms' decisions. On the other hand, each union is aware that the effects of its wage policy on the economy as a whole is negligible. Similarly, each single firm perceives that its decisions on labor and capital input cannot influence the wage setting process because their impact is insignificant relatively to the size of the local labor market. Since the real wage, once negotiated, remains fixed for a certain lapse of time (a "period"), it is reasonable to assume that the wage is set by the union before the realization of the random variable that is relevant for that period.

In this decentralized wage setting, the local union operating in $i$ chooses $v_{it}$ in order to maximize
\[ E_t [u_{it}^k + (1 - \sigma)U_{it}^k] . \]

In each period the union has full control only over the current wage, if we maintain that current union membership cannot commit the workers who will manage the union in the future to the pursuit of policies not optimal from their own temporal perspective. In other words, a wage policy is feasible only if it is time consistent. Hence, the union's problem of choosing the wage in order to maximize (15) can be decomposed into a sequence of similar problems that can be solved recursively.
**Market-determined wages**

In the alternative institutional setup, we have market-determined wages.

Market forces simultaneously determine \( v_{it} \) and \( S_{it} \) in such a way that one has or \( v_{it} > w_t \) entailing \( S_{it} = M_{it} \), or \( S_{it} < M_{it} \) entailing \( v_{it} = w_t \).

Similarly, market forces simultaneously determine \( e_{it} \) and \( A_{it} \) in such a way that one has or \( e_{it} > e_{it}^{\text{min}} \) entailing \( A_{it} = L_{it} \), or \( A_{it} < L_{it} \) entailing \( e_{it} = e_{it}^{\text{min}} \).

**A summary of the timing of events**

Summarizing, in each \( t \) we have a sequence of events in the following order:

i) a new cohort is born; ii) the unions set the wage rates (when the wages are union determined), the unskilled workers decide whether to invest in order to participate actively in the labor market; iii) idiosyncratic shocks occur; iv) firms atomistically determine their demand for skilled workers and apprentices, effective labor supply and wages are determined (when the wages are not union determined), production takes place and apprentices are trained on the job; v) firms reimburse the principal and pay the interest on the capital borrowed at the end of the previous period, firms also pay the dividends to the shareholders, new capital is borrowed by the firms for carrying out production in the next period; investors decide what fraction of their income to save; vi) a fraction \( \sigma \) of each group of population dies at the end of the period, while the surviving individuals decide where to locate in the next period and possibly move into another location.

**3. CHARACTERIZATION OF AN EQUILIBRIUM PATH UNDER UNION-DETERMINED WAGES**

*Equilibrium conditions in the markets for product and physical capital*

Considering (2), (3), (9) and (14), one can easily derive the conditions for equilibrium in the product market:
\[ Y_t + (1 - \delta)K_t = K_{t+1} + C_{it}^{\text{in}} + C_{it}^{\text{sk}} + C_{it}^{\text{un}}, C_{it}^{\text{sk}} = \int_0^n C_{it}^{\text{sk}} \, dt, \int_0^n C_{it}^{\text{un}} \, dt, \quad (16a) \]

and in the market for productive assets:

\[ K_{t+1}^d = K_{t+1}^i, \quad (16b) \]

where \( K_{t+1}^d \) are the assets held by the investors and \( K_{t+1}^i \) are the assets demanded by the firms at the end of \( t \) for production in \( t+1 \).

**Optimal firms’ policies under union-determined wages**

Given the perfectly transferable nature of the general skills acquired by an apprentice, each employer is aware that there is no guarantee that a newly hired worker will remain with his/her firm in the future. This is why an employer does not consider the future returns accruing from the on-the-job training of an apprentice: since the forthcoming benefit of adding a skilled worker to the stock of human capital available to the economy as a whole cannot be appropriated privately, the employer can ignore it as an insignificant externality. Therefore, the selection of the optimal labor policies by a firm amounts in each \( t \) to solving the static decision problem of maximizing (2) with respect to \( S_{it} \) and \( A_{it} \). Given its optimal labor policies, a firm is able to determine at the end of \( t-1 \) the amount of \( K_{it} \) to borrow and install. As wages are union determined, it may be the case that the aggregate demand for either trained labor or apprentices by firms in location \( i \) is rationed. In the aggregate, it is always the case that

\[ S_{it} \leq M_{it}, \quad (17a) \]

and

\[ A_{it} \leq L_{it}. \quad (17b) \]

When labor demand happens to be rationed, it is reasonable to assume that the scarce supply of labor is evenly distributed among firms of the same location. Note that the union wages are not determined at the firm level and that employers cannot
compete for labor in short supply by raising the relevant wages in order to keep and
poach workers, even if skills are perfectly transferable among firms (see Soskice
1990). Therefore, with one as the normalized number of firms of location \( i \), we can
take (17) to be the constraints faced by each individual firm as the union wages induce
all the available skilled and trainable workers to accept a job offer. Hence, the firm’s
choice of the labor inputs amounts to solving the static decision problem of
maximizing (2) subject to (1) and (17).

The resulting optimal labor policies are:

\[
\Sigma_{it} = S(\xi_{it}, M_{it}, K_{it}, \bar{a}_{it}) = \begin{cases} 
\bar{a}_{it} \frac{\o}{1 - (1 - a)} & \text{if } \xi_{it} > \frac{\bar{a}_{it}}{\o} \frac{M_{it}}{a} \\
M_{it} & \text{otherwise,}
\end{cases} 
\]

(18a)

and

\[
\Lambda_{it}^a = \Lambda (\xi_{it}, M_{it}, K_{it}, \o_{it}, \bar{a}_{it}, \bar{a}_{it}) = \begin{cases} 
\frac{m_{it}}{a} W_{it} \frac{\o_{it}}{1 - (1 - a)} & \text{if } \xi_{it} > \frac{\bar{a}_{it}}{\o} \frac{M_{it}}{a} \\
0 & \text{otherwise,}
\end{cases} 
\]

(18b)

The firms' net profits are a strictly increasing function of \( x_{it} \). In fact, using (1), (2)
and (18), we have that:

\[
\pi^a_{it} = \pi^a (x_{it}, M_{it}, K_{it}, s_{it}, v_{it}) - [1 + r_t] - (1 - \delta)] K_{it},
\]

(19)
Given (19), the representative firm operating in i can determine its demand for capital at the end of t by satisfying the optimality condition

\[
\pi^x(\cdot) = \begin{cases} 
(1 - \alpha)M_nk_n \left( \frac{\alpha \alpha_n}{\nu_n} \right)^{\alpha(1-\alpha)} & \text{if } x_n \leq \frac{v_nk_{it}^{\alpha-1}}{\alpha} \\
x_n^{1-\alpha}M_n - M_nv_n & \text{if } \frac{\nu_nk_{it}^{\alpha-1}}{\alpha} < x_n \leq \frac{\mu\nu_nk_{it}^{\alpha-1}}{\alpha\Omega} \\
x_n^{1-\alpha}M_n \left[ 1 + \Omega s_n \right] - v_nM_n - \mu v_n s_n M_n & \text{otherwise.}
\end{cases}
\]

This optimality condition defines \( k_{it+1} \), i.e., the physical capital to skilled labor ratio in the firms of i, as an implicit function of the union’s wage, the trainable labor/skilled labor ratio of i and the market interest rate:

\[
f(k_{it+1}, s_{it+1}, v_{it+1}) = r_{t+1} + \delta, f_1 < 0, f_2 > 0 \text{ and } f_3 < 0,
\]

\[
E_t \left( \frac{\partial \pi^x(x_{it+1}, M_{it+1}, k_{it+1}, s_{it+1}, v_{it+1})}{\partial k_{it+1}} \right) = [(1 + r_{it}) - (1 - \delta)].
\]

Equilibrium in the market for trainable labor and optimal unions’ policy

Having the optimal demand for skilled labor in (18a), we can compute the probability of a skilled worker located in i (before the realization of \( x_{it} \)) being employed in period t:

\[
p(v_{it}, k_{it}) = 1 - \frac{v_{it}k_{it}^{\alpha-1}}{n\alpha(2 - \alpha)}.
\]
By using (22), we can write the period utility expected (before the realization of $x_{it}$) by a skilled worker located in $i$:

$$u^{sk}(v_{it}, k_{it}, w_t) = \left[ 1 - \frac{v_{it}k_{it}^{\alpha - 1}}{n\alpha(2 - \alpha)} \right] u(v_{it}) + \frac{v_{it}k_{it}^{\alpha - 1}}{n\alpha(2 - \alpha)} u(w_t). \quad (23)$$

Similarly, we can use (18b) to compute the probability that a trainable worker located in $i$ (before the realization of $x_{it}$) will be hired in period $t$:

$$q(v_{it}, k_{it}, s_{it}) = 1 - \frac{\mu v_{it}^{\alpha - 1}\left[(1 + \Omega s_{it})^{2 - \alpha} - 1\right]}{n\alpha(2 - \alpha)\Omega^2 s_{it}}, \quad q_1 < 0, q_2 > 0 \text{ and } q_3 < 0. \quad (24)$$

Note that $q(.)$ diminishes as a larger number of inexperienced workers actively search a job, remaining constant both the size of the skilled workforce and the stock of capital located in $i$. In equilibrium, the number of inexperienced workers who decide to search for a job in location $i$ must be such that an inexperienced worker is indifferent between searching a job or staying at home:

$$h(c) = q(v_{it}, k_{it}, s_{it})\left[u(\mu v_{it}) - u(w_t) + (1 - \sigma)(U^{sk}_{it} - U^{un}_{it})\right], \quad (25a)$$

where along an equilibrium path

$$U^{un}_{it} = \beta u(w_t) + \phi U^{un}_{it+1}, \quad \phi \equiv \beta(1 - \sigma). \quad (25b)$$

The period utility function of a skilled worker depends on the real wage and on the physical capital/skilled labor ratio, which is a predetermined variable when a local union sets the wage. Given the forward looking behavior of firms and unskilled workers captured by (21) and (25), the current wage policy of the union cannot affect the union’s future policy and the utility of its members. As a consequence, a local union must solve the following sequence of static problems

$$\max_{v_{it}} u^{sk}(v_{it}, k_{it}, w_t), \quad (26)$$
from which we obtain the following sequence of first-order conditions:

$$ \frac{\partial u^{sk}(v_{it}, k_{it}, w_{t})}{\partial v_{it}} = 0, $$

(27)

defining implicitly the time-invariant wage rule

$$ v_{it} = v(k_{it}, w_{t}), \quad v_1 > 0, \quad v_2 > 0. $$

(28)

Using (28), one can obtain the equation governing the equilibrium path of the lifetime well-being of a skilled worker:

$$ U^{sk}_t = \beta u^{sk}(k_{t+1}, v(k_{t+1}, w_{t}), w_{t}) + \phi U^{sk}_{t+1}, $$

(29)

where the subscript denoting the location is dropped. Indeed, an equilibrium pair \( \{s^*_t, k^*_t\} \), which satisfies (21), (25), (28) and (29) for exogenously given trajectories of \( r_t \) and \( w_t \), and which converges to a steady state \( \{\bar{s}, \bar{k}\} \) (the bar denotes the steady-state value of a variable as wages are union determined), depends on structural parameters assumed to be equal across locations. Therefore, different locations display equal physical capital/skilled labor and trainable labor/skilled labor ratios even if they are endowed with different stocks of physical capital and skilled labor. Hence, local unions are induced to set the same wage in all locations and workers can be indifferent among locations, enjoying the same well-being everywhere. Thus, it is reasonable to assume that at the beginning of each period the skilled workers are evenly distributed across locations:

$$ M_{it} = M_i / n, \quad \forall t > 0. $$

(30)

Note that the unskilled workers are indifferent with respect to the choice of a location because the utility level that they can reach by undertaking some home activity is equal in all locations, and the expected lifetime well-being of an unskilled worker
conditional on his/her active participation in the job market is made equal in all locations to this minimum level of utility by the action of market forces.

**Investors’ optimal policy and general equilibrium path**

For given time profiles of \( r_t \) and \( w_t \), the equilibrium path of \( s_t \) and \( k_t \) is governed by the following system of difference equations:

\[
\Psi(s_{t+1}, k_{t+1}, w_{t+1}, s_t, k_t, w_t) = 0 \tag{31a}
\]

and

\[
f(k_{t+1}, s_{t+1}, v(k_{t+1}, w_{t+1})) = r_{t+1} + \delta, \tag{31b}
\]

where

\[
\Psi(\cdot) = \frac{h(c)}{\phi q(v(k_t, w_t), k_t s_t)} \left[ u(\mu v(k_{t+1}, w_{t+1})) - u(w_{t+1}) \right] - \frac{u^{sk}(v(k_{t+1}, w_{t+1}), k_{t+1}, w_{t+1})}{\phi} - \frac{h(c)}{q(v(k_{t+1}, w_{t+1}), k_{t+1}, s_{t+1})} + u(\mu v(k_{t+1}, w_{t+1}))
\]

is obtained by using (25) to eliminate \( U_{t+1}^{sk} \) and \( U_{t+1}^{sk} \) in (29).

To characterize a general equilibrium path of this economy, one must determine the time profile of the interest rate by solving the optimizing problem of the representative investor. The investors’ optimal plan must satisfy

\[
\theta(1 + r_{t+1}) g'(C^\text{in}_{t+1}) = g'(C^\text{in}_t) \tag{32a}
\]

and

\[
\lim_{t \to \infty} \theta^t K_t g'(C^\text{in}_t) = 0, \tag{32b}
\]

where from (1), (2), (3), (4), (16), (19), (24) and (30) we have that along an equilibrium path
\[ C_{i}^{in} = \frac{M_t}{n} C(v(k_t, w_t), k_t, s_t, k_{t+1}), \]  

(33a)

\[
C(.) = \left[ v(k_t, w_t) \right]^{\frac{\alpha - 1}{2}} k_t^{\alpha - 1} \left[ 1 + \left( \frac{\mu}{\Omega} \right)^2 (1 + \Omega s_t)^{2-\alpha} \left( \frac{\mu}{\Omega} \right)^2 \right] + \frac{k_t^{1-\alpha} n^2}{2} (1 + \Omega s_t)^{\alpha} - 
\]

- \[ n v(k_t, w_t) (1 + \mu s_t) + (1 - \delta) M_t \left[ 1 + s_t g(v(k_t, w_t), k_t, s_t) \right] M_{t+1}. \]

and

\[ M_{t+1} = (1 - \sigma) M_t \left[ 1 + s_t g(v(k_t, w_t), k_t, s_t) \right]. \]  

(33b)

Note that in economies characterized by the same ratios \( s_t \) and \( k_t \), \( C_{i}^{in} \) depends on the scale of production, i.e., on the endowment of skilled labor \( M_t \). Note also that if an acceleration of economic growth generates an increase in firms’ future profits (and future dividends), the resulting wealth effect would tend to boost current consumption, thus reducing the resources necessary to feed the growth process, in the absence of an offsetting movement of the real interest rate. Higher interest rates are required in order to induce the rentiers to increase their savings so as to meet the higher investment demand associated with the acceleration of growth.

We have from (31b), (32a) and (33a) that for equilibrium in the capital market the following condition must hold:

\[
f(k_{t+1}, s_{t+1}, v(k_{t+1}, w_{t+1})) = \frac{g'}{\theta g'} \left( \frac{M_{t+1}}{n} C(v(k_{t+1}, w_{t+1}), k_{t+1}, s_{t+1}, k_{t+2}) \right) n + \delta. \]  

(34)

For commonly used specifications of the investors’ utility function, the equilibrium trajectories of \( k_t \) and \( s_t \) are independent of the scale of the formal segment.
of the economy, i.e., they do not depend on the size of the skilled workforce. For instance, if we let
\[ g(C^m_t) = \left( \frac{C^m_t}{1-\zeta} \right), \quad 0 \leq \zeta < 1, \]  
condition (34) can be rewritten as
\[ \Phi(k_{t+2}, s_{t+1}, k_{t+1}, w_{t+1}, s_t, k_t, w_t) = 0, \]  
where \( \Phi(.) = f(k_{t+1}, s_{t+1}, v(k_{t+1}, w_{t+1})) - \)
\[ \frac{[1 + s_t q(v(k_t, w_t), k_t, s_t)] \zeta [C(v(k_{t+1}, w_{t+1}), k_{t+1}, s_{t+1}, k_{t+2})] \zeta}{\tau (1 - \sigma) \zeta [C(v(k_t, w_t), k_t, s_t, k_{t+1})] \zeta} + 1 - \delta. \]

Therefore, if the wage is union determined, the investors’ utility function is given by (35) and the monetized value of undertaking some home activity is supposed to be fixed at \( w \), a general equilibrium path of \( k_t \) and \( s_t \) must satisfy (31a), (32b) and (36), where \( w_{t+1} = w_t = w \).

**Balanced growth path**

Steady-state values of \( k_t \) and \( s_t \) can be found by setting \( k_t = k_{t+1} = k_{t+2} \) and \( s_t = s_{t+1} \) in (31a) and (36). Indeed, a steady-state pair \( (\bar{k}, \bar{s}) \) must satisfy the system
\[ \Psi(k, s) = 0, \]  
\[ \Phi(k, s) = 0, \]  
where (37a) must hold in order to ensure long-term equilibrium in the market for trainable labor, and (37b) must hold in order to ensure long-term equilibrium in the market for (physical) capital.

Equation (37a) implicitly defines \( s \) as an increasing function of \( k \) (see fig. 1):
\[ s = a(k), \quad a' > 0. \]  

29
Given the number of skilled workers existing in the economy, more physical capital is necessary to induce an increasing number of unskilled workers to participate in the labor market. Other things being equal, a larger number of unskilled workers searching for a job as apprentices depresses any single unskilled worker’s probability of being hired. Thus, this larger s needs to be accommodated by a higher capital stock, which entails both a higher probability of being hired and better lifetime prospects for any single worker if hired.

In contrast, equation (37b) implicitly defines s as a function of k that can be increasing or decreasing in its argument:

\[ s = b(k), \quad b' \geq 0. \]  

(38b)

This ambiguity depends on the forces acting on the two sides of the capital market. On the demand side, a rise in s has a positive effect on the firms’ expected profits: at any level of the capital stock, the increment in expected profits due to a marginally higher k increases with s. Thus, firms demand more physical capital at any given interest rate as s becomes larger. On the supply side, a larger s has a positive wealth effect on investors, since it boosts future growth and profits. Thus, at any given rate of return on capital, investors are willing to consume more and devote fewer resources to capital accumulation when they expect a larger s. Combining demand and supply forces, it follows straightforwardly that a larger s pushes up the equilibrium rate of return on capital, while the overall effect on k is ambiguous. If the investors’ preference for smoothing consumption over time is relatively strong (i.e., if their coefficient of relative risk aversion, \( \zeta \), tends to be large), they require a large increase in r in order to increase saving in the face of improved growth prospects. Hence, the rise in s will depress k, but to an extent that is not sufficient to undermine the improvement in the
growth prospects. Therefore, in this case \( b' < 0 \). The opposite case holds if the investors' period utility is close to increase linearly in consumption (relatively small \( \zeta \)).

It is evident from fig. 1 that \( b' < 0 \) is sufficient to have a unique steady-state pair \((\bar{k}, \bar{s})\). We assume here that this condition holds.\(^{22}\) By linearizing the system consisting of (31a) and (36) about its steady state, one can check that the linearized system typically exhibits saddle-path stability.\(^{23}\) For any initial condition \( k_0 \) in a neighborhood of \((\bar{k}, \bar{s})\), the linearized system characterizes a unique path converging to it.

\(^{22}\) The case in which multiple steady-states are possible is considered in section 6.

\(^{23}\) As a numerical example, if we let \( h(c) = .027577, u(v_t) = v_t \) and \( u(w) = w = .5, n = 1 \), \( \Omega = \mu = .5, \sigma = .01, \alpha = 2/3, \beta = .80808, \tau = .97954, \zeta = .71, \delta = .01238 \), we get \( \bar{k} = 3.138, \bar{s} = .2 \) and \( \bar{v} = .9006815 \). Using the fact that

\[ k_t = k(v_t) = \left[ \frac{2v_t - w}{n\alpha(2 - \alpha)} \right]^{(1 - \alpha)} \]

one can rewrite (31a) and (36) as a system of difference equations in \( s_t \) and \( v_t \). Linearizing the system thus obtained around \((\bar{s}, \bar{v})\), one can derive the following characteristic equation of the linearized system:

\[ \lambda^3 - 3.1862072\lambda^2 + 3.3441234\lambda - 1.1571009 = 0 \]

where \( \lambda_1 = .8834503, \lambda_2 = 1.2775535 \) and \( \lambda_3 = 1.0252034 \) are the solving characteristic roots, implying saddle-path stability.
As $k_t$ and $s_t$ reach their steady-state values, employment and output follow their balanced growth path. Along this path, employment and output grow at their steady-state rate, which is determined only by the parameters of the model:

$$\bar{\rho}_Y = \bar{\rho}_{S+A} = (1 - \sigma)[1 + \bar{s}q(v(\bar{K}, \bar{w}, \bar{s}, \bar{K}, \bar{w})) - 1$$

where $\rho_Y = \frac{Y_{t+1} - Y_t}{Y_t}$, $\rho_{S+A} = \frac{S_{t+1} + A_{t+1} - (S_t + A_t)}{S_t + A_t}$,

$S_t + A_t = M_t[p(v(k_t, w), k_t) + s_tq(v(k_t, w), k_t, s_t)]$

$Y_t = M_t\left\{\frac{C(v(k_t, w), k_t, s_t)}{n} + k_t(1 - \sigma)[1 + s_tq(v(k_t, w), k_t, s_t)] - k_t(1 - \delta) + v(k_t, w)p(v(k_t, w), k_t) + \mu v(k_t, w)s_tq(v(k_t, w), k_t, s_t)\right\}$ and

$M_{t+1} = M_t(1 - \sigma)[1 + s_tq(v(k_t, w), k_t, s_t)].$

Along a balanced growth path, the interest rate increases with the growth rate of the economy, since a higher permanent rate of growth is associated with a higher rate of return on capital:
The fact that along both a transition path and along a balanced growth path the rate of growth does not depend on the scale of the economy has the important implication that economies with identical parameters’ values (structurally similar) but with different initial endowments of skilled labor and physical capital may differ permanently with respect to their levels of $M_t$, $K_t$ and output.\textsuperscript{24} Moreover, if the population grows at the same rate in both economies, their per capita output will not converge over time.\textsuperscript{25} Finally, steady-state wages and probabilities of employment for both skilled and trainable workers are equal in unionized economies sharing the same parameters values even if their employment rate will never converge. Note that this occurs with the wage-setting process taking place independently in each economy (it is not the degree of centralization in wage bargaining that matters, but rather the fact that structurally and institutionally similar economies converge to the same steady-state value of $k_t$ even if their endowments of skilled labor and physical capital remain different). As an implication, there is no incentive for workers located in the poorer economy to migrate to the richer economy, which is the economy where the steady-state proportion of the total population consisting of skilled workers is higher.

\textsuperscript{24} Even along the transition path, two economies with identical parameters but endowed with different initial stocks of skilled labor and physical capital can grow at an identical rate if their initial physical capital-skilled labor ratios are the same.

\textsuperscript{25} Consistently with the model, one can rule out the possibility that the steady-state rate of growth of the skilled population is higher than the steady-state rate of growth of the working population by endogeneizing the birth rate of the working population, i.e., by assuming that in the long run it responds to economic conditions and accommodates $\bar{p}_M$, and/or (if the “economy” does not coincide with the world economy) by allowing inter-country flows of unskilled workers.

\[
1 + \bar{r} = \frac{(1 - \sigma)^{\bar{z}} [1 + \bar{z}q(r(\bar{k}, w), \bar{k}, \bar{z})]^{\bar{z}}}{\theta} = \frac{(1 + \bar{p}_Y)^{\bar{z}}}{\theta}.
\]
An exercise of comparative dynamics

Among other possible experiments, it may be of interest to verify the effects on the steady-state rate of growth of a change in \( \mu \), namely in the fraction of the experienced workers’ wage that firms pay to their newly hired employees.

To analyze these effects we consider the capital market. Using (36), (37b) and (38a), one sees that the steady-state value of \( k \) must satisfy

\[
\frac{f(k, a(k), v(k, w)) - \delta}{\theta} = \frac{(1 - \sigma)^{\bar{\gamma}} [1 + a(k)q(v(k, w), k, a(k))]^{\bar{\gamma}}}{\theta} - 1, \tag{41}
\]

where for optimality on the demand side of the capital market we must have

\[
f(k, a(k), v(k, w)) - \delta = r, \tag{42}
\]

and for optimality on the supply side of the capital market we must have

\[
\frac{(1 - \sigma)^{\bar{\gamma}} [1 + a(k)q(v(k, w), k, a(k))]^{\bar{\gamma}}}{\theta} - 1 = r, \tag{43}
\]

(see fig.2). Equation (41) states the condition that must be satisfied for long-run equilibrium in the capital market only in terms of \( k \) by considering only those values of \( s \) that are consistent for given values of \( k \) with long-run equilibrium of the trainable-labor market.

First we consider only the direct impact of an increase in \( \mu \) on the long-run equilibrium of the capital market: that is, we ignore the impact that the change in \( \mu \) exerts on the capital market through its effects on the long-run equilibrium of the trainable-labor market. On the demand side of the capital market, the larger \( \mu \) will depress future profitability by increasing the cost of trainable labor, thus inducing firms to reduce \( k \) at any given level of \( s \) and \( r \) (the demand curve moves leftwards in fig.2). Together with the fact that it is less convenient for firms to hire new workers, this tends to lower the rate of growth of the economy. By itself, the prospect of a slowing economy depresses future investors’ dividends relatively to current dividends.
Hence, investors are induced to save more at any given interest rate in order to preserve their preferred time-profile of consumption (the supply curve moves rightwards in fig. 2). As a result, the two curves will intersect at a lower steady-state rate of interest associated with a lower growth path.

**FIGURE 2**

Let us now consider the impact exerted by a larger \( \mu \) on the capital market by affecting the combinations of \( k \) and \( s \) consistent with long-run equilibrium on the trainable-labor market.

Other things being equal, a higher entry-wage reduces the chances of a trainable worker being hired, and therefore of acquiring the requisite skills. This makes the participation in the labor market less attractive. On the other hand, if the worker is lucky enough to be hired, s/he can enjoy higher pay in her/his first period of job tenure. This makes participation in the labor market more attractive. The former
effect tends to dominate, especially if workers do not discount the future too heavily, and if they give weight in their choices to the better prospects that they can enjoy during their lifetime once they have acquired the requisite skills on the job. However, one cannot rule the possibility that the latter effect may predominate.

If the depressing effect of a larger $\mu$ on labor market participation predominates, steady-state equilibrium on the trainable-labor market requires that fewer workers participate in the labor market for any given stock of physical capital and size of the skilled workforce. This negatively affects expected firms’ profitability, thereby causing a further leftward movement of the demand curve in fig. 2. Other things being equal, a reduced endowment of trainable workers depresses the growth prospects of the economy, thus influencing investors’ behavior and determining a further rightward movement of the supply curve in fig. 2.

In contrast, if the larger $\mu$ has a boosting effect on labor market participation, more trainable workers will be consistent with long-run equilibrium of the labor market for given stocks of skilled labor and physical capital. This can partially offset the direct impact that the change in $\mu$ has on the capital market, thus limiting the reduction in the steady-state rate of growth brought about by the rise in the apprentices’ relative wage.

4. CHARACTERIZATION OF AN EQUILIBRIUM PATH WHEN THE WAGE DETERMINATION IS COMPETITIVE

Equilibrium wage rates and optimal firms’ policies

Employment of skilled labor in location i is given by substituting $w_i$ for $v_{it}$ in (18a). Similarly, employment of trainable labor in location i is given by substituting
$e_{it}^{\text{min}}$ for $\mu_{it}$ in (18b). Wage rates consistent with competition in the markets for skilled and trainable labor are the following:

$$v_{it} = \begin{cases} 
    w_t & \text{if } x_{it} < \frac{w_t}{\alpha k_{it}^{1-\alpha}} \\
    \alpha x_{it} k_{it}^{1-\alpha} & \text{if } \frac{w_t}{\alpha k_{it}^{1-\alpha}} \leq x_{it} < \frac{e_{it}^{\text{min}}}{\alpha \Omega k_{it}^{1-\alpha}} \\
    \frac{e_{it}^{\text{min}}}{\Omega} & \text{if } \frac{e_{it}^{\text{min}}}{\alpha \Omega k_{it}^{1-\alpha}} \leq x_{it} < \frac{e_{it}^{\text{min}} (1 + \Omega s_{it})^{1-\alpha}}{\alpha \Omega k_{it}^{1-\alpha}} \\
    \alpha x_{it} k_{it}^{1-\alpha} (1 + \Omega s_{it})^{1-\alpha} & \text{otherwise},
\end{cases}$$

(42a)

and

$$e_{it} = \begin{cases} 
    e_{it}^{\text{min}} & \text{if } x_{it} < \frac{e_{it}^{\text{min}} (1 + \Omega s_{it})^{1-\alpha}}{\alpha \Omega k_{it}^{1-\alpha}} \\
    \alpha \Omega x_{it} k_{it}^{1-\alpha} (1 + \Omega s_{it})^{1-\alpha} & \text{otherwise},
\end{cases}$$

(42b)

where $\frac{e_{it}^{\text{min}}}{\Omega} \geq w_t$. Skilled workers’ wages may even fall to the minimum wage required to induce a skilled worker to accept a job offer when the local shocks are adverse, while a rise in them is bounded by the competition of inexperienced workers when shocks are more favourable. Indeed, when the shock is favourable enough to make it convenient for firms to employ not only all the available skilled workers but also some trainable workers $x_{it} > \frac{e_{it}^{\text{min}}}{\alpha \Omega k_{it}^{1-\alpha}}$, market forces operate so as to determine wage rates at which firms are indifferent between hiring a skilled worker or hiring an apprentice. The apprentices’ wage rises above $e_{it}^{\text{min}}$ to eliminate excess demand for apprentices only when local shocks are highly favourable and even the
The entire trainable workforce is employed \( x_{it} \geq \frac{e_{it}^{\min} (1 + \Omega s_{it})}{\alpha \Omega k_{it}^{1-\alpha}} \), thus allowing also \( v_{it} \) to increase together with \( e_{it} \).

Firms’ net profits are now given by

\[
\pi^n = \pi^f (x_{it}, M_{it}, k_{it}, s_{it}, v_{it}) - (r_i + \delta)K_{it} \cdot \pi^f(\cdot) = (1 - \alpha)M_{it} k_{it} \left( \frac{1}{v_{it}} \right)^{\frac{\alpha}{1-\alpha}} ,
\]

(43)

where \( v_{it} \) is given in (42a).

The representative firm operating in i is able to determine its demand for capital at the end of t by satisfying the optimality condition (20), where the skilled workers’ wage is now given in (42a) and the firms’ gross profit function is given in (43). This optimality condition defines \( k_{it+1} \) as an implicit function of the reservation wage of the trainable workers located in i, of the trainable labor/skilled labor ratio of i, and of the market interest rate:

\[
z(k_{it+1}, s_{it+1}, e_{it+1}^{\min}, w_{t+1}) = (r_{t+1} + \delta), \quad z_1 < 0, \quad z_2 > 0, \quad z_3 < 0 \quad \text{and} \quad z_4 < 0,
\]

(44)

where

\[
z(\cdot) = \frac{1 - \alpha}{2n\alpha(2 - \alpha)K_{it+1}^{2-\alpha}} \left[ \left( \frac{e_{it+1}^{\min}}{\Omega} \right)^2 \left[ 1 - (1 + \Omega s_{it+1})^{2-\alpha} \right] - w_{t+1}^2 \right] + \frac{(1 + \Omega s_{it+1})^{\alpha} (1 - \alpha)n}{2k_{it+1}^{\alpha}}.
\]

**Skilled workers’ utility and unskilled-labor market equilibrium**

The period utility expected (before realization of \( x_{it} \)) by a skilled worker located in i is the following:

\[
u^{sk}(k_{it}, s_{it}, e_{it}^{\min}, w_{t}) = \bar{E}_t [u(v_{it})], \quad u^{sk}_s > 0, \quad u^{sk}_s < 0, \quad u^{sk}_{e_{it}} > 0, \quad u^{sk}_{w_{it}} > 0,
\]

(45)

where \( v_{it} \) is given by (42a).

Given the stock of physical capital installed by the i firms, the expected lifetime utility of a trainable worker located in i tends to decrease with the number of
inexperienced workers on search within the area. This number must therefore adjust so as to equalize the utility loss associated with the search activity to the differential in lifetime well-being that an individual may expect if s/he participates in the labor market instead of staying at home:

\[ h(c) = \hat{E}_t[u(e_{it})] - u(w_t) + (1 - \sigma)(U_{it}^{sk} - U_{it}^{un}), \quad (46a) \]

where, considering that \( e_{it} \) is given by (42b), one has

\[ \hat{E}_t[u(e_{it})] = e(k_{it}, s_{it}, e_{it}^{\text{min}}), \quad e_k > 0, e_s < 0, e_{e_{it}^{\text{min}}} > 0. \quad (46b) \]

Given (10) and (50b), one can rewrite (50a) as

\[ h(c) = e(k_{it}, s_{it}, e_{it}^{\text{min}}) - e_{it}^{\text{min}}, \quad (46c) \]

which implicitly defines \( e_{it}^{\text{min}} \) as a function of \( k_{it} \) and \( s_{it} \):

\[ e_{it}^{\text{min}} = m(k_{it}, s_{it}), \quad m_k > 0, m_s < 0. \quad (46d) \]

Along an equilibrium path, the lifetime expected utility of an unskilled worker is still governed by (25b), while

\[ U_{it}^{sk} = \beta u^{sk}(k_{it+1}, s_{it+1}, m(k_{it+1}, s_{it+1}), w_{t+1}) + \phi U_{t+1}^{sk}, \quad (47) \]

where the subscript denoting the location is dropped. Indeed, an equilibrium pair \( (k^*, s^*) \), which satisfies (25b) and (44)-(47) for exogenously given trajectories of \( r_t \) and \( w_t \), and which converges to \( (\hat{k}, \hat{s}) \) (the hat denotes the steady-state value of a variable as the wages are competitively determined), depends only on structural parameters that are invariant across locations. Given the workers’ indifference to locations, it is still assumed that at the beginning of each period the skilled workers are evenly distributed across locations as in (30).

**General equilibrium path**

For given time profiles of \( r_t \) and \( w_t \), the equilibrium path of \( s_t \) and \( k_t \) is governed by the following system of difference equations:

\[ \Lambda(s_{t+1}, k_{t+1}, w_{t+1}, s_t, k_t, w_t) = 0 \quad (48a) \]

and
\begin{equation}
\zeta(k_{t+1}, s_{t+1}, m(k_{t+1}, s_{t+1}), w_{t+1}) = r_{t+1} + \delta,
\end{equation}

where
\begin{equation}
\Lambda(\cdot) = h(c) + u(w_t) - e(k_t, s_t, m(k_t, s_t)) \frac{\phi}{\phi} - u^k(k_{t+1}, s_{t+1}, m(k_{t+1}, s_{t+1}), w_{t+1}) - h(c) + e(k_{t+1}, s_{t+1}, m(k_{t+1}, s_{t+1}))
\end{equation}
is obtained by using (25b) and (46a) to eliminate \( U_{t+1}^k \) and \( U_{t+1}^k \) in (47).

A general equilibrium path of this economy with competitive determination of wages can be characterized, by using the investors’ optimality conditions (32), where now along an equilibrium path
\begin{equation}
C_t^{\text{in}} = \frac{M_t}{n} D(k_t, s_t, m(k_t, s_t), k_{t+1}, w_t),
\end{equation}

\begin{equation}
D(\cdot) = \left[ \zeta(k_t, s_t, m(k_t, s_t), w_t) + (1 - \delta) \right] \ln \frac{1}{1 + s_t q(m(k_t, s_t), k_t, s_t)} \ln k_{t+1}
\end{equation}
and
\begin{equation}
M_{t+1} = (1 - \sigma) M_t \left[ 1 + s_t q(m(k_t, s_t), k_t, s_t) \right].
\end{equation}

Note that \( q(\cdot) \), which is the fraction of the trainable workforce hired in \( t \), is obtained by substituting \( t_{\min} \) for \( \mu_i \) in (24).

Considering the optimality conditions for demand and supply of capital, equilibrium in the capital market implies
\begin{equation}
\Pi(k_{t+2}, k_{t+1}, s_{t+1}, w_{t+1}, k_t, s_t, w_t) = 0,
\end{equation}
where
\begin{equation}
\Pi(\cdot) = \zeta(k_{t+1}, s_{t+1}, m(k_{t+1}, s_{t+1}), w_{t+1}) - \left[ 1 + s_t q(m(k_t, s_t), k_t, s_t) \right]^{\tau} \left[ D(k_{t+1}, s_{t+1}, m(k_{t+1}, s_{t+1}), w_{t+1}, k_{t+1}) \right]^{\gamma} + 1 - \delta.
\end{equation}

Therefore, if the wage is determined competitively, the investors’ utility function is given by (35) and the monetized value of the workers’ outside option is fixed at \( w \), a general equilibrium path of \( k_t \) and \( s_t \) must satisfy (32b), (48a) and (50), where \( w_{t+1} = w_t = w \). Note that along this path the dynamics of \( k_t \) and \( s_t \) do not depend on the size of the skilled workforce \( M_t \) or on stock of physical capital \( K_t \).
Balanced growth path

A steady-state pair \((\hat{k}, \hat{s})\) must satisfy the system

\begin{align*}
\Lambda(k, s) &= 0, \\
\Pi(k, s) &= 0,
\end{align*}

where (51a) must hold if long-term equilibrium in the market for trainable labor is to be ensured, and (51b) must hold if long-term equilibrium in the market for (physical) capital is to be ensured.

Equation (51a) implicitly defines \(s\) as an increasing function of \(k\):

\[ s = i(k), \quad i' > 0. \]

As under union-determined wages, the equation that any combinations of \(k\) and \(s\) must satisfy in order to ensure steady-state equilibrium in the capital market implicitly defines \(s\) as a function of \(k\) which may be increasing or decreasing in its argument:

\[ s = n(k), \quad n' \geq 0. \]

This ambiguity has the motivations discussed in the previous section, and we restrict our attention to the case \(n' < 0\), which is sufficient to obtain a unique steady-state pair \((\hat{k}, \hat{s})\). Furthermore, by linearizing the system consisting of (48a) and (50) about its steady state, one can check that the linearized system typically exhibits saddle-path stability: for any initial condition \(k_0\) in a neighborhood of \((\hat{k}, \hat{s})\), the linearized system characterizes a unique path converging to it.

As in the previous section, a relatively large \(\zeta\) (coefficient of relative risk aversion) ensures that \(n' < 0\).

As a numerical example, if we take the same parameter values and the same functional form of the workers' utility function used in the example of the previous section (see note 23), one obtains \(\hat{k} = 2\) and \(\hat{s} = .2\). By linearizing the system consisting of (48a) and (50) around this steady-state pair, one can derive the following characteristic equation of the linearized system:

\[ \lambda^3 - 5.9724664\lambda^2 + 6.5505695\lambda - 1.5460953 = 0, \quad \text{where} \quad \lambda_1 = .329616, \]
As \( k_t \) and \( s_t \) reach their steady-state values, employment and output follow their balanced growth path. Along this path, employment and output grow at their steady-state rate \( \dot{\rho}_Y = \dot{\rho}_{S+A} \), which is determined in this case, too, only by the parameters of the model:

\[
\dot{\rho}_Y = \dot{\rho}_{S+A} = (1 - \sigma)[1 + \dot{s}q(m(\hat{k}, \hat{s}), \hat{k}, \hat{s})] - 1.
\]

(53)

At steady state, the interest rate is higher as the economy grows faster:

\[
1 + \hat{\rho} = \frac{(1 - \sigma)\hat{s}[1 + \dot{s}q(m(\hat{k}, \hat{s}), \hat{k}, \hat{s})]^{\frac{1}{\sigma}}}{\theta} = \frac{(1 + \dot{\rho}_Y)^{\frac{1}{\sigma}}}{\theta}.
\]

(54)

It is worth noting that with competitive wages, too, the rate of growth does not depend on the scale of the economy, and thus economies with different initial endowments of skilled labor and physical capital may grow at the same rate forever, and differ permanently with respect to their levels of \( M_t \), \( K_t \) and output. Rather than depending on the modality of wage determination, the permanent influence of the initial conditions stems from the fact that the availability of a large (small) skilled workforce, which is the product of past history, attracts abundant (scarce) physical capital, whose presence induces a large (small) number of individuals to qualify for acquiring skills on the job. However—as we shall see in the next section—this fact by itself is not sufficient to generate hysteresis.

**Competitive determination of wages versus union determination of wages: comparing the balanced growth paths**

At the end of the previous section, we saw how a larger \( \mu \) (which is the fraction of the experienced workers’ wage that firms pay to their newly hired employees under union determination of wages) may lower the steady-state rate of growth of the economy. Thus, it is fair to compare the growth performances of the

\[ \lambda_2 = 1.0131541 \text{ and } \lambda_3 = 4.6296962 \] are the solving characteristic roots, implying saddle-path stability.
economy under competitive wage determination and under union wage determination by assuming that \( \mu \to \Omega \), i.e., by assuming that, with union wage setting, the pay differential between experienced workers and apprentices tends fully to reflect the productivity differential between them. Even under these circumstances, \( e = \Omega \nu(k,v) \) is typically higher than \( e^{\min} = m(k,d(k)) \) at any level of \( k \): since \( e = \Omega \nu(k,v) \) is linked to the skilled workers’ wage that unions negotiate by exploiting their members’ monopoly over the required skills, it tends to be higher than the minimum wage at which the trainable workers are willing to work when wages are determined competitively. Similarly, the skilled workers’ wage is on average lower at any level of \( k \) in the absence of monopoly unions negotiating wages in each location.

To analyze the effects of these differences in expected wage cost on the balanced growth path of the economy, we can use (50), (51b) and (52a) to verify that under competitive determination of wages the steady-state value of \( k \) must satisfy

\[
(z(k,i(k),m(k,i(k)),w) - \delta = \frac{(1 - \sigma)^k [1 + i(k)q(m(k,i(k)),k,i(k))]}{\theta} - 1 = r , \quad (55)
\]

where for optimality on the demand side of the capital market we must have 
\( z(k,i(k),m(k,i(k)),w) - \delta = r \), and for optimality on the supply side of the capital market we must have 
\[ (1 - \sigma)^k [1 + i(k)q(m(k,i(k)),k,i(k))]}{\theta} - 1 = r \] (see the dotted-line curves in fig. 3).

In the spirit of the previous section, for analytical purposes we distinguish between the direct impact that institutional differences in wage determination exert on the long-run equilibrium of the capital market and the impact that they exert on it by affecting the long-run equilibrium of the trainable labor market (by affecting the number of unskilled workers willing to participate in this market). Let us suppose that an unionized economy is subject to a reform which shifts its wage-setting institutions toward competitive wage determination. Along the new \( (k,s) \) schedule that is
consistent with long-run equilibrium of the capital market, firms will demand more capital at any level of $s$ and $r$ because the expected marginal profitability of capital is boosted by the reduction in expected labor cost (the demand curve in fig. 3 shifts rightwards). Other things being equal, this tends to accelerate steady-state growth and to increase future profits relatively to current ones, thereby inducing investors to save less at any level of $s$ and $r$ (the supply curve in fig. 3 shifts leftwards). As a result, the two curves will intersect at a higher steady-state rate of return on capital, which is associated with a higher balanced growth path.

**FIGURE 3**

Next, consider the effect that long-term adjustment in the trainable labor market will have on the steady-state growth rate. As wages respond to competitive forces rather than being union determined, the average entry-wage tends to decline at any given $k$ and $s$. Other things being equal, this raises the number of new jobs created for apprentices. The greater probability of finding a job as apprentice may induce more people to invest in labor market participation even if they expect to receive
lower wages once hired. If this is the case, a lower $k$ is required for long-run equilibrium of the trainable labor market for any given level of $s$. This will strengthen the forces already at work on the two sides of the capital market. In the opposite case, in which the negative influence on labor market participation exerted by lower expected wages predominates over the positive influence determined by the higher probability of finding a job, the adjustments required for long-run equilibrium in the trainable labor market may partially offset the movements of the two curves in the $(K,r)$ plane resulting from the institutional shift toward competitive wage determination. In this case, indeed, the relative scarcity of trainable workers can attenuate the positive impact on expected profitability of lower labor costs. In both cases, however, we end up having $\hat{\rho}_Y > \bar{\rho}_Y$ and $\hat{r} \geq \bar{r}$. a competitive wage-setting process creates a more dynamic environment in which the economy can grow at a perpetual rate higher than that possible under union determination of wages. Economies characterized by the same structural parameters have different long-term growth performances, depending on the their labor-market institutions.

An important caveat to this conclusion is that, in general, a reform which gives rise to competitive wage determination is not Pareto improving: along a balanced growth path the skilled workers (the “insiders” of this economy) are typically better off when wages are union determined.29

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28 We have $\hat{r} = \bar{r} = \theta^{-1} - 1$ if and only if $\zeta = 0$. Taking the numerical examples of note 23 and 27, in which the same parameters’ values were assigned to economies differing for the modalities of wage determination (and in which $\mu \rightarrow \Omega$ under union determination of wages), we have $\hat{\rho}_Y = .0648362 > \bar{\rho}_Y = .0023$ and $\hat{r} = .0782335 > \bar{r} = 0328839$.

29 In the numerical examples of note 23 and 27, we have $\bar{U}^{sk} = 3.1168365 > \hat{U}^{sk} = 2.8848935$. 

45
5. CHARACTERIZATION OF AN EQUILIBRIUM PATH IN A TWO-REGION ECONOMY

Suppose now that the two economies we have studied in autarky open up so that there is a single market for capital and a single market for the only output produced in this two-region world. Both regions share the same parameter values, but they preserve their specific mode of wage determination. Markets for skilled labor remain distinct. This amounts to saying that skilled workers do not move from their own region (for instance, because interregional mobility is relatively costly), while it is immaterial where the investors are located since they can hold assets of both regions.

Consistently with previous assumptions, the large number of identical investors (the firms’ owners) operating in the world economy is normalized to be two. Again, for simplicity and without loss of generality, we assume that all investors are entitled to receive an equal share of the firms’ net profits. The representative investor must decide a contingency plan for \( C^\text{in}_t \) and \( K_{t+1} \) in order to maximize:

\[
E_0 \left[ \sum_{t=0}^{\infty} \theta^t g(C^\text{in}_t) \right],
\]

subject to \( K_{t+1} + C^\text{in}_t \leq (1 + r_t) K_t + \frac{\pi^u_t + \pi^m_t}{2} \), where \( r_t \) is the world interest rate, and the subscripts “U” and “M” denote a variable which belongs—respectively—to the region in which the wages are union determined and to the region in which the wages are market determined.

Equilibrium in the market for productive assets requires:

\[
2K_t = K^u_t + K^m_t,
\]
and the condition for equilibrium in the product market is the following:

\[ Y_{Ut} + Y_{Mt} + (1 - \delta) (K_{Ut} + K_{Mt}) = 2K_{t+1} + 2C^{in}_{t} + C^{sk}_{Ut} + C^{sk}_{Mt} + C^{un}_{Ut} + C^{un}_{Mt}. \]  

(58)

The investors’ optimal plan must satisfy (32), where now along an equilibrium path

\[ C^{in}_{t} = \frac{M_{Ut}}{2n} C(v(k_{Ut}, w), k_{Ut}, s_{Ut}, k_{Ut+1}) + \frac{M_{Mt}}{2n} D(k_{Mt}, s_{Mt}, m(k_{Mt}, s_{Mt}), k_{Mt+1}, w), \]  

(59)

the motion of \( M_{Ut} \) is governed by (33b), the motion of \( M_{Mt} \) is governed by (49b) and \( w_{t+1} = w_{t} = w \).

An equilibrium path of the world economy is characterized by (31), (32b), (33), (48), (49) and by

\[ \frac{g \left( \frac{M_{Ut}}{2n} C(., .) + \frac{M_{Mt}}{2n} D(., .) \right)}{g \left( \frac{M_{Ut+1}}{2n} C(., .) + \frac{M_{Mt+1}}{2n} D(., .) \right)} = r_{t+1} + \delta. \]  

(60)

The world interest rate reflects both the anticipation of (relatively) high future profits generated by the firms located in the more dynamic region and the anticipation of (relatively) low future profits made by the firms located in the low-growth region. During the transition to a balanced growth path, this allows the high growth region to finance its growth process at a cost that is lower than it would be under autarky. Indeed, the anticipation of a lower growth path in the unionized region mitigates the wealth effect, which would exert a stronger upward pressure on investors’ current consumption—and therefore on the interest rate that they require to provide resources for investment—if they had claims only on the profits of firms located in the high...
growth region. As a result, the region with competitive wage determination is able to grow along a transition path at a higher rate than under autarky. The opposite applies to the unionized region.

Given the investors’ utility function in (35), the world interest rate coincides in the long run with the steady-state interest rate characterizing--in autarky--the economy whose wages are determined competitively: \( r^\# = \bar{r} \) (where \( \# \) denotes the steady-state value of a variable in the two-region economy). The intuition is straightforward: in the presence of a permanent differential of growth between the two regions, the share of the investors’ total income generated in each period by the firms located in the low-growth region becomes asymptotically insignificant, no matter what the initial endowments of skilled labor of the two regions may be. Therefore, in the long run, the equilibrium world rate of interest will reflect only the returns obtainable by investing in the high-growth region. As time passes, and as the scale of the high-growth economy grows ever larger relatively to the scale of the unionized economy, the world capital market is increasingly dominated by the investment conditions prevailing in the high-growth economy.

An important implication of the tendency of the world interest rate increasingly to reflect the better opportunities for investment existing in the more dynamic economy is that the steady-state rate of growth of the region with union-determined wages is lower with an integrated capital market than it is under autarky: \( \rho^\#_{Y_U} = \rho^\#_{A_U+S_U} < \bar{\rho}_Y = \bar{\rho}_{A+S} \). The reason should again be intuitive: the higher cost of capital--due to the fact that on the demand side of the capital market there are firms located in a region in which profitability is higher--discourages investment by firms located in the region with union-determined wages. In its turn,
this lower investment activity induces fewer individuals to participate in the labor market, with a further depressing effect on growth.

6. POSSIBILITY OF MULTIPLE BALANCED GROWTH PATHS

The strategic complementarity between physical capital and trainable labor creates the possibility of multiple balanced growth paths. This is the case both in the presence of unions and with competitive wage determination.

Under union wage determination, indeed, a pair \( \left( s_t, \frac{1}{s_t} \right), \left( v_t, \frac{1}{v_t} \right) \) satisfying (31) for given trajectories of \( r_t \) and \( w_t \) need not be unique. In this case, the anticipation of a larger supply of trainable workers may increase the firms’ expected profitability, thus boosting the firms’ demand for capital in the economy as a whole, in spite of the higher union wage resulting from the more favourable trade-off between wage and probability of employment faced by the skilled workers due to the increased capital accumulation. In other words, the expected marginal profitability of (physical) capital may not decrease with a larger \( k_{t+1} \) because the more abundant supply of trainable labor reduces the probability of a shortage of apprentices when local circumstances are favourable and firms want to expand production. In its turn, the more favourable trade-off between probability of employment and wage leads to a higher expected utility for the trainable workers, thereby inducing a larger number of unskilled workers to participate actively in the formal labor market. This validates firms’ optimistic expectations. The opposite may happen if firms anticipate a smaller supply of trainable workers. In both cases, however, firms’ expectations may be fulfilled because they give rise to an increase (or a decrease) in the capital stock which induces
both the local unions and the unskilled workers to behave consistently with those expectations.

Under competitive wage determination, the potential for multiple balanced growth paths depends on the fact that the minimum wage at which the newly hired workers are willing to accept a job tends to decrease when the economy is expected to move along a higher growth path. In this case, in fact, the differential in lifetime well-being between remaining an unskilled workers and acquiring the requisite skills is larger. Moreover, the increased number of workers who decide to participate in the labor market at any expected entry-wage moderates the apprentices’ wage in the case of favourable local shocks. This decline in trainable labor cost along a higher growth path prevents the expected marginal productivity of capital from falling as the capital-skilled labor ratio increases. Thus, firms’ expectations of higher profitability along a growth path characterized by more investment in capital may be self-fulfilling precisely because of the increased participation in the trainable labor market that results from the rise in investment.

No matter what the modalities of wage determination are, the potential for multiple balanced growth paths cannot actualize itself if the equilibrium rental rate of capital is very sensitive to changes in capital demand. A higher growth path cannot be sustainable if the rise in interest rate required by the investors to meet the demand for the additional investment necessary to accelerate the growth process is too high relative to the increment in discounted profits generated by boosting growth. In general, in fact, the interest rate must be higher to meet a larger demand for physical capital in order to stimulate investors to hold more productive assets. This is especially true if we consider that a higher growth path is associated with an increase in the expected sequence of discounted dividends paid to investors, and thus reduces
the amount of saving that they are willing to provide at any level of the interest rate. Therefore, the potential for multiple equilibrium paths is able to actualize itself only if the investors’ preference for smoothing consumption over time is weak (\(\zeta\) close to 0), so that they are willing to finance higher growth for a modest increase in the rate of return on capital. In the limiting case in which \(\zeta=0\) (constant marginal utility of consumption), different permanent rates of growth are consistent with the same equilibrium rate of return on capital. If the rate of return on capital required by the investors remained unchanged as capital investment increases, there would be a larger differential in permanent rates of growth between a high growth and a low growth steady state. In these circumstances, in fact, there is no upward pressure on the cost of capital as firms demand more capital: no increase in the cost of capital can dampen the potential of a permanently higher rate of growth created by the positive externalities generated by the investment activity.

It is evident from the previous discussion why the existence of multiple balanced growth paths requires, under union determination of wages, that \(b' > 0\) for at least some range of values of \(k\): along the curve giving the combinations of \(k\) and \(s\) consistent with the long-run equilibrium of the capital market, there must be an interval of values of \(k\) within which an increment of \(s\) increases the expected marginal profitability of capital more than the rate of return required by the investors. It is possible to verify that this is actually the case when multiple steady-state pairs of \(k\) and \(s\) exist (see figures 4 and 5). Typically, we can have two steady-state pairs

\[n^* > 0\] for at least some range of values of \(k\). The following discussion on multiple balanced growth paths applies also to an economy with competitive determination of wages.
(\bar{K}^h, \bar{s}^h) and (\bar{K}^l, \bar{s}^l), such that \bar{K}^h > \bar{K}^l and \bar{s}^h > \bar{s}^l.\textsuperscript{31} For the case in which \(\zeta > 0\), numerical examples show that, linearizing the system consisting of (31a) and (36) around these steady states, the linearized system characterizes a unique path of \(k_t\) and \(s_t\) converging to \((\bar{K}^h, \bar{s}^h)\) for any given \(k_0\) in a neighborhood of \(\bar{K}^h\), while it characterizes a continuum of paths converging to \((\bar{K}^l, \bar{s}^l)\) for any given \(k_0\) in a neighborhood of \(\bar{K}^l\).\textsuperscript{32} For the case in which \(\zeta = 0\), the system governing \(k_t\) and \(s_t\) reduces to a single first-order difference equation, the motion of \(k_t\) and \(s_t\) is completely governed by forward-looking expectations (the initial condition on \(k_0\) does not play a role in determining the dynamics of \(k_t\) and \(s_t\)), and the system can

\textsuperscript{31} As numerical examples, let \(h(c) = 9.2576516 \times 10^{-3}\), \(u(v_t) = v_t\) and \(u(w) = w = 1\). \(\Omega = .25\), \(\mu = .2681247\), \(\sigma = .03\), \(\alpha = .75\), \(\beta = .4333714\), \(\tau = .8617753\), \(\zeta = .025\), \(n = 1\), \(\delta = .28255\), we get \(\bar{K}^h = .1\), \(\bar{s}^h = .2\), \(\bar{K}^l = .0988142\), \(\bar{s}^l = .1\) and \(\bar{p}^1 = -.0106306\). Letting \(h(c) = .0123832\), \(u(v_t) = v_t\) and \(u(w) = w = 1\). \(\Omega = .25\), \(n = 1\), \(\mu = .2681247\), \(\sigma = .03\), \(\alpha = .75\), \(\beta = .4906097\), \(\tau = .8247422\), \(\zeta = 0\), \(\delta = .2290823\), we get \(\bar{K}^h = .1\), \(\bar{s}^h = .2\), \(\bar{K}^l = .0986489\), \(\bar{s}^l = .1\) and \(\bar{p}^1 = -.0106358\).

\textsuperscript{32} In the first numerical example of the preceeding note, the characteristic equation of the system obtained by linearizing (31a) and (36) around \((\bar{K}^h, \bar{s}^h)\) is the following: \(\lambda^3 - 8.6241583\lambda^2 + 7.5331318\lambda + .1914909 = 0\), from which one can obtain the characteristic roots \(\lambda_1 = -.0247185\), \(\lambda_2 = 1.0147784\) and \(\lambda_3 = 7.6340984\); while the characteristic equation of the system obtained by linearizing (31a) and (36) around \((\bar{K}^l, \bar{s}^l)\) is the following: \(\lambda^3 - 9.0816481\lambda^2 + 7.6891839\lambda + .2878336 = 0\), from which we can obtain the characteristic roots \(\lambda_1 = -.0359049\), \(\lambda_2 = .9858373\) and \(\lambda_3 = 8.1317157\).
immediately jump to one of its two long-run equilibria, or it can follow one among a continuum of paths converging to \((\bar{k}^1, \bar{s}^1)\) in a neighborhood of it \(^{33}\) (see fig. 6).

\[ \begin{align*}
\text{FIGURE 4} \\
\end{align*} \]

\[ b(k) \]

\[ a(k) \]

\[ \bar{k}^1 \]

\[ \bar{k}^h \]

\[ \bar{s}^h \]

\[ \bar{s}^1 \]

\[ k \]

\[^{33}\text{In the second numerical example of note 31, (31a) can be rewritten as a first-order difference equation in } k_t \text{ only, by using the fact that (36) implicitly defines } s_t \text{ as a function of } k_t. \text{ Linearizing this difference equation around } \bar{k}^h \text{ yields the following characteristic equation: } \lambda - 1.0210701 = 0, \text{ from which one can obtain the characteristic root } \lambda = 1.0210701; \text{ while the characteristic equation obtained by linearizing (31a) around } \bar{k}^1 \text{ is the following: } \lambda - .9812596 = 0, \text{ from which one can obtain the characteristic roots: } \lambda = .9812596. \]
It is worth noting that the rate of growth of employment and output is permanently higher at \((\bar{k}^h, \bar{s}^h)\) than it is along the balanced growth path associated
with \((\bar{K}^1, \bar{s}^1)\): \(\bar{p}^1_Y = \bar{p}^{b^h}_{S+A} > \bar{p}^{l^h}_Y = \bar{p}^{l^h}_{S+A}\). The associated steady-state interest rates are: \(\bar{r}^h \geq \bar{r}^l\). \(^{34}\)

Given the cumulative nature of the growth process, the levels of output and employment of an economy moving along a path characterized by \((\bar{k}^h, \bar{s}^h)\) diverge over time from the employment and output levels of a structurally similar economy starting with equal initial endowments \(K_0\) and \(M_0\) but moving along a path characterized by \((\bar{K}^1, \bar{s}^1)\). This implies that in the presence of multiple balanced growth paths, the «animal spirits» of capital-market participants may determine irreversible long-lasting effects on economic performances. If capital-market participants are convinced that the growth path characterized by a high rate of capital accumulation is not sustainable because firms’ profitability will be too low to guarantee a sufficiently high rate of return on a larger amount of productive assets, their pessimism will prove self-fulfilling. Indeed, low capital investment will reduce workers’ investment in labor-market participation, thus depressing firms’ expected profitability and inducing the economy to move along a low-growth equilibrium path: real interest, real wage and employment will be lower than they would be in a high growth scenario. The notion itself of «natural» rates of interest and unemployment loses its significance in this context.

Finally, it should be emphasized that a high-growth equilibrium path is always Pareto superior than a low growth path. Indeed, in a high growth regime i) investors enjoy a higher rate of return on their accumulated wealth, ii) the skilled workers are able to exploit a more favourable trade-off between real wage and the probability of

\(^{34}\) Again, one has \(\bar{r}^h = \bar{r}^1 = \theta^{-1} - 1\) if and only if \(\zeta = 0\).
being employed, thus increasing their expected lifetime sequence of discounted utilities, iii) the apprentices’ wages are higher, iv) a larger number of workers have the opportunity to be trained on the job and increase their human wealth, and v) the lifetime prospects of an unskilled worker remain unchanged.

7. FISCAL TRANSFERS AND CONVERGENCE

The intention now is to show that once the rigidity in the value of the workers’ outside option is removed, namely once \( w_t \) is sensitive to general economic conditions, the mechanism generating persistence (on-the-job-training) is unable to prevent initial differences in endowments of reproducible assets between economies similar in any other respect from fading away in the long run. The most natural way to do so is by assuming that the workers’ outside option depends on fiscal transfers in support of jobless households.

**Autarky**

Fiscal transfers must be financed by collecting taxes:

\[
\frac{Tax_t}{N_t - (A_t + S_t)},
\]

where \( Tax_t \) are the total tax revenues in period \( t \) and \( N_t \) is the total number of workers in period \( t \). Assuming that all the workers without jobs devote their entire income to consumption, the product-market equilibrium condition can be rewritten as

\[
Y_t + (1 - \delta)K_t = K_{t+1} + C^\text{in}_t + C^\text{sk}_t + C^\text{un}_t + [N_t - (A_t + S_t)]w_t.
\]

The tax base is the income on wealth that is not reinvested (capital investment is tax deductible), namely that portion of the investors’ income devoted to consumption. Therefore, in the case of union-determined wages, we have
\[
\text{Tax}_t = \gamma \frac{M_{t+1}}{n} C(v(k_t, w_t), k_t, s_t, k_{t+1}), \quad 0 < \gamma < 1, \quad (63)
\]

where \(\gamma\) is the tax rate and \(C(\cdot)\) is given in (33a). An economy with a low employment rate is both an economy in which the tax base for financing the welfare benefits paid to the jobless is reduced and an economy in which there is a large number of households to support with fiscal transfers. The combination of these two effects inevitably decreases the value of the workers’ outside option.

To model the dynamics of \(w_t\), we must use the laws of motion of \(N_t\) and \(A_t + S_t\). The total population of workers evolves according to:
\[
N_{t+1} = (1 - \sigma + \xi)N_t, \quad \xi > 0, \quad (64)
\]
where \(\xi\) is the birth rate, while the employed workforce evolves in an unionized economy according to:
\[
S_t + A_t = M_t[p(v(k_t, w_t), k_t) + s_t q(v(k_t, w_t), k_t, s_t)], \quad (65)
\]
where the dynamics of \(M_t\) is given in (33b).

Therefore, in an unionized economy, (61) can be rewritten as
\[
w_t = \frac{\gamma d_t C(v(k_t, w_t), k_t, s_t, k_{t+1})}{n[1 - d_t[p(v(k_t, w_t), k_t) + s_t q(v(k_t, w_t), k_t, s_t)]]} - d_t = \frac{M_t}{N_t}, \quad (66a)
\]
where \(d_t\) evolves according to
\[
d_{t+1} = d_t \frac{(1 - \sigma)[1 + s_t q(v(k_t, w_t), k_t, s_t)]}{1 - \sigma + \xi}, \quad d_0 \text{ given}. \quad (67a)
\]

Adding equations (66a) and (67a) to (31a) and (36) yields the extended system in \(k_t\), \(s_t\), and \(d_t\) governing the evolution of the unionized economy when \(w_t\) is determined endogenously. Note that \(k_t\), \(s_t\), and \(d_t\) determine the dynamics of the employment rate (emprate\(_t\)), since

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Similarly, under competitive wage determination, (61) can be rewritten as

\[
\text{emprate}_t = d_t \left[ p(v(k_t, w_t), k_t) + s_t q(v(k_t, w_t), k_t, s_t) \right], \quad \text{emprate}_t = \frac{A_t + S_t}{N_t}. \quad (68a)
\]

Adding equations (66b) and (67b) to (48a) and (50) yields the extended system in \( k_t, s_t \), and \( d_t \) governing the evolution of the economy under competitive wage setting when \( w_t \) is determined endogenously. In this case, we have

\[
\text{emprate}_t = d_t \left[ p(w_t, k_t) + s_t q(m(k_t, s_t), k_t, s_t) \right]. \quad (68b)
\]

Once \( w_t \) is allowed to respond to general economic conditions, the workers living in an economy which inherits lower stocks of physical capital and skilled labor from the past have poorer alternative options if they remain out of work than do the workers of a region that has inherited from the past higher stocks of physical capital and skilled labor. Wage determination will be affected by this difference in the value of the workers’ outside option. An adjustment process, therefore, takes place which induces economies characterized by the same structural and institutional features but with different initial endowments of skilled labor and capital to converge in the long run to the same output level, «natural» rate of employment and value of the workers’ outside option.
(66a), (67a), (31a) and (36), one can obtain a steady-state triple \((k^*, s^*, d^*)\), which implies that under union determination of wages the steady-state employment rate, \(e^{mprate^*}\), depends only on the parameter values of the economy, and that the steady-state rate of growth of the economy coincides with the rate of growth of the workers’ population. Similarly, in the case in which wages are determined competitively, one can obtain a steady-state triple \((k^o, s^o, d^o)\), which implies that the steady-state employment rate, \(e^{mprate^o}\), depends only on the parameter values of the economy, and that the steady-state rate of growth of the economy coincides with the rate of growth of the workers’ population. However, one can check that \(e^{mprate^o} > e^{mprate^*}\): steady-state levels of output per capita and employment rate (the natural employment rate) are higher in an economy with competitive wage determination than they are in an unionized economy. Indeed, as the process of wage determination is dominated by the insiders, \textit{ceteris paribus} the wage pressure is stronger. Therefore, the value of the outside option must be lower in order to keep labor cost under control and to stimulate firms to invest at a rate which allows the economy to grow at the same rate as the workers’ population \((w^o > w^*)\). Given the tax rate \(\gamma\), the value of the outside option is lower if and only if the level of output per capita and the employment rate are lower.

Another important implication of the existence of fiscal transfers in favour of the jobless is that--given the modalities of wage determination--a higher tax rate tends to depress the natural employment rate: \(\frac{\partial e^{mprate^*}}{\partial \gamma} < 0\) and \(\frac{\partial e^{mprate^o}}{\partial \gamma} < 0\). As a larger share of total resources is devoted to supporting the jobless, the steady-state rate of employment must decline in order to prevent \(w^*\) (or \(w^o\) in the case of competitive wage determination) from increasing and pushing up the wages. Higher wages, indeed, would be inconsistent with that rate of capital of capital investment required to
make the economy grow at the same rate as the workers’ population. Since both \( w^* \) and \( w^\circ \) do not change with \( \gamma \), one can conclude that—given the modalities of wage determination—a steady state associated with a lower \( \gamma \) is Pareto superior with respect to a steady state associated with a higher \( \gamma \). In fact, at the steady state associated with a lower \( \gamma \), fewer workers are jobless, investors’ consumption is higher, while the employed and the jobless workers have the same income. However, along a transition path, workers can be worse off when the share of resources devoted to support the jobless is reduced because in the short term this will lower the value of their outside option.

**Two-region economy**

Suppose that two regional economies sharing the same parameters’ values and the same wage-setting institutions are fully integrated and that investors’ consumption is taxed by the national government in order to pay equal benefits to all the jobless no matter in which region they are located. For simplicity and without loss of generality, we assume that the two regions have the same initial population \( (N_{N0} = N_{S0} = N_0) \) and physical capital to skilled labor ratio \( (k_{N0} = k_{S0} = k_0) \), where the subscripts «N» and «S» denote, respectively, the «northern» region and «southern» region of this two-region economy.

Consistently with section 5, the equations for equilibrium in the market for productive assets and in the product market can be rewritten, respectively, as

\[
2K_t = K_{Nt} = K_{St} \tag{69}
\]

and

\[35\]

Given this assumption on the initial physical capital-skilled labor ratio, it is not only at steady state but also along a transition path that no worker is willing to migrate into the other region even if the cost of moving is negligible.
\[ Y_{N_t} + Y_{S_t} + (1 - \delta)(K_{N_t} + K_{S_t}) = 2K_{t+1} + 2C_{t+1}^{in} + C_{N_t}^{sk} + C_{S_t}^{sk} + C_{N_t}^{un} + C_{S_t}^{un} + (2N_t - A_{N_t} - A_{S_t} - S_{N_t} - S_{S_t})w_t. \] (70)

In an unionized economy, each workless household receives a net benefit given by

\[ w_t = \frac{\gamma(d_{N_t} + d_{S_t})C(v(k_t, w_t, k_t, s_t, k_{t+1}))}{n[2 - (d_{N_t} + d_{S_t})][p(v(k_t, w_t, k_t) + s_t q(v(k_t, w_t), k_t, s_t))]}, \] (71)

where \( d_{N_t} \) and \( d_{S_t} \) evolve, respectively, according to

\[ d_{N_{t+1}} = d_{N_t} \frac{(1 - \sigma)[1 + s_t q(v(k_t, w_t, k_t, s_t))]}{1 - \sigma + \xi}, \] \( d_{N_t} \) given, \hspace{1cm} (72a)

and

\[ d_{S_{t+1}} = d_{S_t} \frac{(1 - \sigma)[1 + s_t q(v(k_t, w_t, k_t, s_t))]}{1 - \sigma + \xi}, \] \( d_{S_t} \) given. \hspace{1cm} (72b)

Adding equations (72) to (31a) and (36) yields the extended system in \( k_t, s_t, d_{N_t} \) and \( d_{S_t} \) governing the evolution of the two-region unionized economy when \( w_t \) is determined endogenously.

Since there is a unique steady-state value of the outside option that is consistent with employment and output growing at the same rate as population does, one can easily check that the natural rate of employment in the North (South) is a decreasing function of the natural rate of employment in the South (North):

\[ \frac{\partial \text{emprate}_{N}^{*}}{\partial \text{emprate}_{S}^{*}} < 0. \] Indeed, given the tax rate \( \gamma \), lower levels of output and employment in the South (North) shrink the tax base and increase the number of households in need of government support, thereby exerting nationwide a downward pressure on the
value of the workers’ outside option, which allows the North (South) to raise production and employment without pushing up the wages. A similar analysis applies also to the case where wages are determined competitively.

8. SUMMARY OF THE MAIN RESULTS

The model presented in this paper has four main implications:

i) Structurally similar economies sharing the same modalities of wage determination may display permanent differentials in per capita output and employment rate. The presence of wage-negotiating unions has been shown to be irrelevant with respect to the tendency of economies characterized by the same structural features, i.e., by the same parameter values, forever to have different levels of per capita output and employment rates if their initial endowments of physical capital and skilled labor differ. The fact that a mechanism causing persistence (on-the-job-training) is at work is a necessary but not sufficient condition for this tendency to come about. Indeed, for there to be permanent disparities among structurally and institutionally similar economies (full hysteresis) it is crucial that jobless workers achieve an utility level which is immune to movements in the volume of production and employment. In other words, there is an absence of convergence in the levels of output and employment when the monetized value of the workers’ alternative option does not respond to changes in output and employment. In this case, there is no incentive for workers to migrate into the richer economy. Conversely, as the workers’ outside opportunities are affected by the overall performance of the economy, structurally and institutionally similar economies characterized by different initial endowments of skilled labor and physical capital converge to the same level of output per capita and
to the same «natural» rate of employment, i.e., steady-state fraction of the working-age population that is employed.

ii) However, as the monetized value of the workers’ alternative options is rigid and does not respond to general economic conditions, it is shown that the modes of wage determination are able to affect the long-term growth path of the economy (and the employment level) by influencing expected profitability—and therefore the incentive to firms to invest—and workers’ willingness to participate in the labor market: other things being equal, the steady-state rate of growth is higher when wages are determined competitively rather than being negotiated by unions. Assessing this result, it should be noted that—in general—institutional reforms which shift the wage-setting process from union determination of wages to competitive wage determination are not Pareto-improving: insiders prefer to have wages negotiating by their unions. Moreover, when the workers’ outside opportunities depend on fiscal transfers which respond to general economic conditions, the natural rate of employment to which an economy converges is higher when wages are determined competitively, and—other things being equal—when the tax rate is lower and a reduced share of total resources is devoted to supporting the jobless. Finally, in the presence of a central government paying equal benefits to all jobless households living in a two-region economy, the natural employment rate of a region is higher—other things being equal—when the lower is the natural rate of employment of the other region.

iii) The anticipation of an acceleration of growth causes a positive wealth effect due to the expectation of higher future firms’ profits, thus pushing current savings down. In the context of a closed economy, where the resources for financing the additional investment necessary to fuel growth can be augmented only by increasing domestic savings, this implies that to move the economy along a higher growth path the rate of
return on savings must increase so as to dominate the wealth effect and stimulate saving. Typically, the positive permanent effect exerted on the growth rate by some favourable shift in labor-market institutions or in the market psychology will be partially offset by a rising equilibrium cost of capital: a permanently higher growth rate is associated with a higher steady-state rate of interest. In the context of an open economy, this constraint is less binding, and the growth potentialities of an economy enjoying a competitive advantage due to the modalities of wage determination can be better exploited at the expenses of economies less able to attract investment because of their labor-market institutions. Indeed, in the presence of an integrated world capital market, investment in a region which tends to grow more can be financed at least partially from abroad by attracting resources from regions in which profitability is expected to be lower, thereby raising the cost of capital of the latter and determining a larger differential between their long-term growth performances. Therefore, as the modalities of wage determination vary across regions, the interregional integration of the capital market permanently reduces the rate of growth of the region offering the worst prospects to investors because of the role played by the unions in wage determination. This is because the cost of capital (the world interest rate) tends asymptotically to equalize the rate of return on capital that the region characterized by the highest profitability and growth rate is able to offer to investors. In such a region, the possibility to draw from the world capital market can accelerate growth only temporarily, since—in the long run—the moderating influence on the world interest rate of the lower rate of return on investment prevailing in the less competitive regions tends to vanish.

iv) When the workers’ outside option is rigid, the current state of the «fundamentals» may not allow determination of a unique growth path consistent with a sustainable
macroeconomic equilibrium: given two structurally and institutionally (sharing the same modalities of wage determination) similar economies having the same current endowments of physical capital and skilled labor, there may be a differential in growth performance in favour of the economy whose growth potentialities are evaluated more optimistically by those participating in the capital market. In this context, in fact, the «average opinion» of participants in the capital market may play a role in the «selection» of one among the multiple equilibrium paths along which an economy may move, while the notion of a «natural» interest rate loses its significance, since identical economies may move permanently along balanced growth paths associated with different steady-state rates of interest.
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