FINANCIAL MARKET IMPERFECTIONS,
HETEROGENEITY AND GROWTH

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Financial Market Imperfections, Heterogeneity and Growth

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Abstract

This paper offers a model of growth with heterogeneous agents in which, due to asymmetric information, financial markets do not work properly. In such a world, the Modigliani-Miller theorem fails to hold, a financial hierarchy emerges and ‘how to finance’ the engine of growth – in our case represented by uncertain endeavours in R&D - matters. In turn, heterogeneity means that agents lack sufficient information on the behaviour adopted by the others, forcing them to make use of naïve rules in forming expectations and in calculating their probability of bankruptcy. The basic properties of the model are explored via simulations. In particular, it is possible to appreciate how a worsening of financial conditions (e.g. an increase of the contractual interest rate on loans or of the probability of bankruptcy) affects negatively the long-run average rate of growth.

Keywords: Growth, R&D, Heterogeneity, Financial Market Imperfections.

JEL Classification: E44, O16.

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1. Introduction

Growth theorists have always made a large use of ‘stylized facts’ to guide the direction of their research agenda. In fact, since Kaldor’s fist attempt to list them, the number of what scholars think are relevant stylized facts of economic growth has sensibly increased. This enlargement of the set of relevant regularities has mirrored successful efforts in the “... creative art of construction of new models” (Romer, 1994, p.11). In particular, new ideas on how to model long-run growth have emerged by the consciousness – confirmed by an increasing amount of up-to-date statistical evidence - that one of the main implication of the Solow’s theory, namely cross-country convergence, lacks of any empirical support. Indeed, the theoretical view received at the beginning of the ‘80s was not able to reconcile the fact of a large number of poor countries trapped in underdevelopment equilibria with low per-capita income, investment and technical progress, with the fact that capital does not flow (much) from rich to poor countries.

Endogenous growth theory has changed our way of thinking about growth by providing a picture made of multiple self-fulfilling expectations, multiple equilibria and development thresholds. We claim that whether this task has been accomplished is a result of ‘positive spillovers’ with a contemporaneous ongoing paradigmatic shift in macroeconomics reasoning: the New-Keynesian stream of thought\(^1\), with its emphasis on coordination failure due to strategic complementarities and positive spillovers (Cooper and John, 1988), and on the failure of the First Fundamental Theorem of Welfare Economics in the presence of imperfect competition (Blanchard and Kiyotaki, 1987). Admittedly, much less attention has been paid by growth theorists to the other main message launched out by New-Keynesian Economics (NKE), that is an urge to find an explicit role for asymmetric information as a cause of imperfections on financial markets\(^2\).

Admittedly, informational imperfections in financial markets have long be recognized as one of the most important reasons why levels of income and rates of growth between the developed and less developed economies

\(^1\) Dixon and Rankin (1995) prefer to talk of New Macroeconomics, as a “… shift towards a macroeconomics based on microfoundations with market imperfections of one kind or another” (p.1).

\(^2\) For a critical assessment of this two distinct lines of research of New-Keynesian Economics see Ardeni et al. (1993).
do not converge\(^3\). Nevertheless, models with endogenous growth in general do retain the implicit assumption of perfect capital markets. In other words, the accumulation of reproducible factors – either physical or human capital - can be always financed, the only constraint being given by a *no-Ponzi-game* transversality condition. Given suitable contour conditions for the optimization problem, the system is thus solved for its steady-state rate of growth. In a world with imperfectly competitive markets for goods and labour but perfect information, this is not a problem. However, once we allow for the presence of asymmetric information, firm’s ability to get money from potential shareholders or bank loans is severely hampered by the adverse selection and the adverse incentive effects: equity and credit rationing can occur as equilibrium phenomena. In this case, investment and production decisions are strictly constrained by the availability of funds\(^4\).

In this paper we investigate the functioning of an imperfectly competitive model of growth, where internal funds are not sufficient to finance both production and R&D investments, and external finance is costly due to asymmetric information between borrowers and lenders. The financing issue displays major effects on the long-run behaviour of our model economy. In particular, simulations confirm that a worsening of financial conditions (due e.g. to an increase of the contractual interest rate on loans or to an increase of the probability of bankruptcy) decreases the long-run average rate of real growth. In this sense, our model could be inscribed in the small but growing literature which aims to explore the nexus between financial markets and growth, which we briefly survey in section 2.

The remainder of the paper is organized as follows. In section 3 we discuss some general features of an imperfectly competitive growth model where financial issues matter. Such a model is then formalized in section 4. In section 5, we analyze some simulated examples in order to assess the impact of financial variables - notably, the interest rate on loans and the bankruptcy cost – on the dynamic behaviour of our model economy. Section 6 concludes.

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\(^3\) See e.g. Stiglitz (1989).

\(^4\) A survey of the economics of imperfect capital markets from a New-Keynesian perspective is provided by Delli Gatti and Tamborini (1998).
2. Related literature

Development economists have long recognised the tight relationship between growth and financial markets conditions, as an explanation for the disappointing performance of less-developed countries\(^5\). Far less attention to this topic has been devoted by ‘endogenous-growth’ theorists. Some relevant exceptions exist, however.

In a first generation of endogenous-growth models with an explicit role for the financial sector, research draw attention on the different functions financial intermediaries might play in promoting growth. So, in Greenwood and Jovanovic (1990) and Bencivenga and Smith (1991) the development of financial intermediation fosters real economic growth acting as a device for a better resource allocation\(^6\). This task is accomplished either by pooling risks across investors whenever information on expected rate of returns are not promptly available, or by insuring investors against the risk of unpredictable liquidity needs.

The additional issue of the role played by agency costs due to asymmetric information in weakening economic development is taken up in a second group of papers, e.g. Bencivenga and Smith (1993) and Ma and Smith (1996). Central to these contributions is the interaction between informational asymmetries that give rise to credit rationing (Stiglitz and Weiss, 1981) and real growth, in an overlapping generation model with social increasing returns to scale of the Romer’s (1986) type. In these models, equilibrium – that is, steady-state - growth rate and level of credit rationing are jointly determined and inversely related: the higher the degree of rationing on the credit market, the lower the steady-state rate of real growth.

From a different perspective, Greenwald, Kohn and Stiglitz (1990) and Stiglitz (1992, 1993) stress the role of retained earnings or net worth in financing real growth. The approach followed in these papers does not assign any formal role to the costly state verification problem typical of any agency relationship. Alternatively, net worth matters because managers of firms are wary of debt financing, since this increases their probability of bankruptcy. On its part, this means that managers are risk-averse, risk-aversion being formalized by making their utility a concave function of

\(^5\) See Patrick (1966), McKinnon (1973) and Shaw (1973).
\(^6\) See also the survey by Pagano (1993).
profits. In particular, the relative simplicity of this framework allows their proponents to explore the nexus between business cycles and long-run growth, this latter being driven by learning-by-doing or explicit R&D efforts in productivity enhancing activities. In both cases, imperfections on the capital market means that economic downturns can have damaging long-run consequences. Economic downturns have adverse effects on firms’ cash flow, and are likely to be accompanied by more extensive credit rationing. In turn, less expenditures on R&D and lower levels of investments and production result in less learning. Hence, after a slowdown the growth path of the economy is permanently shifted down.

In such a framework, fluctuations arise endogenously as a co-ordinated self-fulfilling shift of individuals’ expectations – very much in line with the ‘animal spirits’ hypothesis - on future economic conditions. In addition, Delli Gatti and Gallegati (1996) demonstrate that simply inserting a ‘no shirking constraint’ in the wage formation process may give rise to irregular – i.e. chaotic – fluctuations around an endogenous trend.

Our model intends to further elaborate in this direction, by studying the consequences for the ‘risk-aversion’ class of models of treating explicitly the following issues:

1) An imperfectly competitive goods market.

3. An informal discussion of the model.

Our basic framework is a dynamic imperfectly competitive economy, where a given number of heterogeneous firms produce and sell non-perfectly substitutable commodities. A double level of rationality characterizes firms7. First, they resort to routinized behaviour in dealing with the forecast of variables characterized by a remarkable degree of complexity (i.e. forecasts regarding technological change, the behaviour currently adopted by competitors, and the probability to yield negative profits and consequently to incur in bankruptcy). Second, they employ updated relevant variables in the short-run profit maximization program.

In order to survive in an effectively competitive environment, firms are

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7 From this point of view, Meyer, Vogt and Voßkamp (1996) take an approach very similar to the one considered in this paper.
periodically involved in uncertain search activities with the aim of improving their competitiveness by means of technological progress. In our setting, by technological progress we mean technologies whose productivity is higher than the existing ones. Therefore, we abstract from product innovations. The level of effort put forth by a firm is measured by the amount of money spent in technological search processes, which we label Research and Development (R&D).

The economy is sequential, with discrete periods of time - indexed with $t$ - lasting $2\tau$, so that $t+1 = t + 2\tau$. The production cycle of each firm starts at the beginning of period $t$, and it takes $\tau$ to be completed. In $t+\tau$ commodities are brought to the market and sold out. The time schedule of production and marketing implies that firms must be able to pay for their desired input before being able to sell their output. In line with standard New-Keynesian theorizing (see e.g. Greenwald and Stiglitz, 1993), we assume that ex ante informational asymmetries on the stock market prevent firms from issuing new equities. In turn, they are allowed to finance entirely their production by recurring to credit, at an additional cost determined by the risk of bankruptcy (borrower's risk in Keynes' terminology). Bankruptcy occurs whenever profits are less or equal to zero. The total number of firms is on average maintained fixed by allowing new entries to replace exactly the number of exits due to bankruptcy with a one time lag.

At the same time, a two stage lottery starts, which depending on the amount of money invested in it determines: $i$) whether the R&D effort has success, and $ii$) how much productivity increases as a result of successfully R&D. Recall that in the presence of asymmetric information on the capital market, external finance is more costly than internal funds. Among the set of investment opportunities, risk averse firms prefer to finance the more uncertain endeavour by recurring to the cheaper source, that is to available cash-flow. In this model, R&D expenditure - whose impact on productivity retains the higher degree of uncertainty - will be preferably financed by means of retained profits. Furthermore, as emphasised by Stiglitz (1993), investments expenditures in R&D differ from other investments because they are not, in general, collateralizable.
The chart below provides an illustration of the time structure.

<table>
<thead>
<tr>
<th>$t$</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t+\tau$</td>
<td>Commodities are marketed. Firms gain profits and invest them in the R&amp;D lottery.</td>
</tr>
<tr>
<td>$t+2\tau$</td>
<td>The $t+1$ productivity parameter is discovered.</td>
</tr>
</tbody>
</table>

**Figure 1** – Production and R&D in the sequence economy

Summarizing, firms are destined to lose or to gain market shares according to their present ability to make profits, which ultimately depends on their competitive position in the past. The competitive selection process provides an incentive to invest in R&D. At the microeconomic level, entrepreneurs invest basically in order to conserve and expand their competitiveness. At the aggregate level there is competition-led endogenous growth, given that successful R&D allows an increase in the average productivity, thus an expansion in the aggregate productive capacity.

### 4. Analytical details.

One can think of the economy as being restricted to a particular industry composed of a population of firms producing a single good in $K$ varieties. In turn, exclusively one firm produces each variety. A representative consumer endowed with a constant elasticity of substitution (CES) additive utility function of the Dixit-Stiglitz type expresses the following standard demand curve for commodity $j$:

$$
Y^d_j(t) = \left( \frac{P_j(t)}{Q(t)} \right)^{-\sigma} \frac{Y^d}{K},
$$

where $P_j(t)$ represents the price charged by firm $j$, $Y^d$ is the real aggregate income available for consumption in the whole economy, $\sigma$ is the constant
elasticity of substitution between couples of good, with $\sigma > 1$; and finally $Q(t)$ is a CES index for the aggregate price level:

$$Q(t) = \left( \frac{1}{K} \sum_{j=1}^{K} P_j(t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{2}$$

We assume that, besides the difference in the variety of the common good they produce, firms can differ because of the technology they employ, which translates into heterogeneity of relative market shares. Let us assume that firm $j$ production function is $y_j(t) = z_j(t) n_j^a \rightarrow n_j = \frac{y^a}{z^a} \rightarrow n_j = \frac{y^a}{e}$, where $n$ is labour demand, and $z_j$ represents a productivity parameter. Thus, total costs are equal to:

$$C_j(t) = W \frac{Y_j(t)^e}{e_j(t)}, \tag{3}$$

with $\alpha > 1$. Being labour the only production factor, total costs are de facto a wage bill, where $W$ is the nominal wage rate. Expression (3) states that for a given peculiar technology parameter $e_j(t) = \bar{e}_j$, each firm experiences increasing static real marginal costs. Nevertheless, costs can be dynamically decreasing as soon as the variable $e_j(t)$ follows a sufficiently fast increasing path. This is indeed the mechanics of real growth in our model\footnote{In a companion paper (Gaffeo, 1999), the author uses a similar framework to explore the implications of alternative assumptions on the degree of localization of technological change for the long-run average rate of growth.}, to which we now move our attention.

### 4.1 Effective competition as the engine of growth.

In order to understand the long-run dynamics of the model, it is worthy to have a look at a sort of ‘deterministic’ version of the model. As a first-order approximation, let us suppose that the population of heterogeneous firms dwelling in the economy can not pursue any innovative activity. The absence of technological change implies that at least one Nash-type
equilibrium can be easily found. Recall, from equation (1), that the demand function for a generic firm $j$ is given by:

$$Y^d_j(t) = \left( \frac{P_j(t)}{Q(t)} \right)^{-\sigma} D(t,K), \quad (1')$$

where we have substituted for the term $Y(t)/K$ the expression $D(t,K)$. Notice that the latter is equal to the market share of firm $j$ along a symmetric long-run equilibrium, where $P_j = Q \Rightarrow \frac{P_j}{Q} = 1$; in our model, this constitutes a useful benchmark of comparison for the market share a firm can obtain along an heterogeneous equilibrium. For any given level of nominal aggregate demand, $D(t,K)$ is clearly a function of the number of firms operating at time $t$.

The relative competitive position of firm $j$ when adopting its peculiar technology $1/e_j$ is then given by the ratio of firm $j$’s market share and potential market share in the homogeneous case:

$$\frac{Y^d_j(t)}{D(t,K)} = \left( \frac{P_j(t)}{Q(t)} \right)^{-\sigma}. \quad (4)$$

Let us define this competitive ratio as $X_j(t,K)$; hence, $X(t,K)$ is the vector of relative competitive shares for each firm $j = 1, \ldots, K+1$. This represents a complete configuration at each time $t$ of an imperfectly competitive market composed of $K$ firms.

Taking logs, one obtains:

$$x_j(t,K) = -\sigma (p_j(t) - q(t)), \quad j = 1,2,\ldots,K+1, \quad (5)$$

where lower case letters represent the logarithms of the relative variables. Obviously, a positive $x_j$ means that firm $j$ has a market share higher than the one characterizing the symmetric equilibrium; the opposite holds for a negative $x_j$. The system of equations (5) identifies the equilibrium competitive ratio for every firm $j = 1,2,\ldots,K+1$, in a form which closely
resembles the notion of ‘fitness’ as largely employed by evolutionary economics. In this literature, fitness is a useful biological metaphor for the ability of a firm to survive in a competitive environment. In our framework, the lower the price a firm is allowed to fix, the higher its overall fitness, i.e. its relative competitive position.

Armed with these insights, we can now provide a preliminary analysis of what happens in the long-run for three alternative situations: 1) the case in which firms are homogeneous; 2) the case in which firms are heterogeneous, but technology can not change; 3) the case in which the degree of heterogeneity can change as time goes by, because of productivity-enhancing R&D investments.

### 4.1.1 The homogeneous case.

Under the assumption of homogeneity, it immediately follows that the Bertrand-Nash equilibrium solution is defined as $\sum_{j=1}^{K} x_j = 0$. Hence, in this case the equilibrium can be derived as $x^h = \{0,0,...,0\}^T$, where the apex $h$ defines the homogeneous case.

### 4.1.2 The heterogeneous static case.

Suppose now the population of firms is heterogeneous, and that they might be ranked from the more to the less efficient, $e_1 > e_2 > ... > e_K$. When firms employ heterogeneous technologies, at any time $t$ the market is described by the vector $x^n = \{x_1, x_2, ..., x_K\}^T$ (the apex $n$ identifying the heterogeneous case), and there exists a technology $e_j^*$ such that $x_j < 0$ holds if $e_j < e_j^*$, and $x_j > 0$ if $e_j > e_j^*$. This threshold value may be found noting that if one substitutes from the BRF, it follows that:

$$x_j (K) = -\sigma \omega_0 (\omega_1 - \ln e_j) ,$$

\[6\]

---

9 In evolutionary economics, a relative fitness function is defined as the differential equation

$$\frac{ds_i}{dt} = v (E_i - \langle E \rangle) s_i ,$$

where $s$ is the relative market share of firm $i$, $E_i$ its somehow determined competitiveness, and $\langle E \rangle$ the average competitiveness of all firms in the industry ($= \Sigma s, E_i$). For relevant applications of replicator dynamics, see e.g. Iwai (1984) and Silverberg (1988); for interesting discussions on the relation between evolutionary equilibria and equilibrium concepts in economics, see e.g. Friedman (1991) and Joosten (1996).
where $\omega_0 = 1/\delta$, and $\omega_1 = \ln A + (\alpha - 1) \ln D$. Thus, heterogeneity for itself does not sensibly affect the market structure of the long-run equilibrium, given that the degree of polymorphism which initially characterizes the market propagates to its long-run solution.

### 4.1.3. The heterogeneous dynamic case.

More interesting results emerge if we abandon the static representation in order to consider the effect of technological change. Suppose that each firm is engaged in learning-by-doing or on-the-job-training activities, which allow the firm to improve its productivity at a given rate. Now relative prices potentially change at each time. Therefore, we can obtain a replicator function by simply taking the derivative with respect to time of equation (5):

$$
\begin{align*}
\dot{x}_j(t) &= -\sigma \left( \frac{p_j(t)}{p_j(t)} - \frac{q(t)}{q(t)} \right) x_j(t), \quad j = 1, 2, \ldots, K,
\end{align*}
$$

i.e., the rate of change of the competitiveness of firm $j$ as measured by the relative market share increases (decreases) as far as its price grows less (more) than the average price.

Let us suppose that the increase in productivity - and the consequently decrease in price - allowed by investments in R&D is a deterministic function of retained profits, $p_j(t) = h(\pi_j(t))$, with $h' < 0$. In turn, profits are higher the higher is the relative market share of firm $j$, $\pi_j(t) = f(x_j(t))$, $f' > 0$. It follows that $p_j(t) = h'f(x_j(t))$, where the symbol $\sqcap$ indicates the inner product. Recall that at any time $t$ fitness is measured by $\{-\sigma[p_j(t) - q(t)]\}$. It is straightforward to note that the fitness function and the dynamic process representing the evolution of the population composition are sign-compatible, that is they have the same sign whenever $x_j > 0$. Under this condition, we can follow the arguments reported in Joosten (1996) to demonstrate that the deterministic system of $K+1$ equations composed by the $K$ equations in (7) and the relationship for the aggregate price index admits at least one fixed point. Furthermore, every stable fixed point is a saturated equilibrium, that is an equilibrium at which each survived firm has

---

10 As usual, we state the existence of a fixed point for the dynamic system under consideration if there exists at least one trajectory that did not start in it, but which converges to it.
highest fitness. As the dynamic process goes on, firms whose market share prevents them from gaining profits go bankrupt, and the number of operating firms shrinks. Eventually, only the fittest firms will survive.

If we allow investments in R&D to be a stochastic outcome, the dynamics of the system remains seriously affected, in a way which depends primarily on the characteristics of the probability distribution of the random process. As a result, the long-run properties of the system can be in general detected only by making use of simulations. Nevertheless, it is important to stress that even in the stochastic case the incentive to invest in R&D remains as the only way to survive.

To summarize, firms have an incentive to invest in R&D as much as they can, because this is the only means they have to survive. As we said, the expression ‘as much as they can’ assumes here a special meaning: due to asymmetric information on the financial markets, firms investing in R&D are forced to recur exclusively to retained profits, given that external finance is not available.

### 4.2 Production decisions.

At the beginning of time $t$, $\forall t \in T$, $K$ firms play a one-shot game adopting Bertrand-type strategies. The owner of the firm producing variety $j$ determines her optimal price, and via the demand function her optimal output as well, given consumers’ preferences and the aggregate price index. Recall that the aggregate price index measures the average behaviour of his competitors. Commodities are then sold in $t + \tau$.

Since production costs must be incurred before cashing in sales proceeds, firms must pay wages to workers in advance. In other words, they must raise funds to anticipate the wage bill. Due to asymmetric information, the issue of new equities is ruled out: equity rationing occurs at any instant. Furthermore, profits inherited from the past (i.e. from period $t - 1$) are used up to finance R&D investments. Therefore, firms can finance the wage bill in period $t$ exclusively by borrowing from banks. For simplicity, we assume that firms, in financing production, have unlimited access to debt; the banking system accommodates the demand for loans at the given contractual rate of interest. Debt is repaid completely in each period, so that there is no debt accumulation. If at $t + \tau$ real profits becomes negative, firms incur in bankruptcy. Associated to bankruptcy there is a cost to managers, that is assumed to be an increasing function of output.
Thus, the problem consists in maximizing profits for period $t$ with respect to the relative price $\frac{P_j(t)}{Q(t)}$:

$$\max E[\tau_j(t) - BC_j(t)] = \left( \frac{P_j(t)}{Q'(t)} \right)^{1-\sigma} D(t,K) - (1 + r) \left[ \frac{1}{\epsilon_j} \left( \frac{P_j(t)}{Q'(t)} \right)^{-\sigma} D(t,K) \right] - d \left( \frac{P_j(t)}{Q'(t)} \right)^{1-\sigma} D(t,K) \phi_j \tag{8}$$

where $BC_j(t)$, i.e. bankruptcy costs, is assumed to be an increasing linear function of the proceeds of sales, depending on the probability of bankruptcy $\phi_j$; $d$ is the marginal bankruptcy cost; $r$ is the contractual interest rate on loans. The real wage is assumed to be equal in every period to 1; in other words, we assume that a real wage equal to 1 clears the labour market at each $t$.

From an inspection of the maximization problem (8), one can easily realize that our framework departs from the ones usually analyzed in the literature for two relevant issues.

First, notice that firm $j$ neither knows the actual prices of competitors, nor the aggregate price index, which will become revealed information only in $t + \tau$. This is an obvious application of the informational problem arising in non-competitive decentralized economies: firms set their prices simultaneously given their private information, but price decisions interact since firms compete over market shares. While in the standard literature on monopolistic competition this problem is solved by assuming a homogeneous Nash equilibrium (each entrepreneur knows the price chosen by the others, since it is equal to the one she chooses), in heterogeneous markets competitors’ prices have to be somehow forecasted. Given that firms change continuously their technologies, and that each firm has a positive but strictly less than one probability to find an innovation, the environment is a stochastic, non-stationary one. A sketch of the proof is provided in the appendix. From non-stationarity it follows that the notion of rational expectations can not be applied\textsuperscript{11}. As an alternative, simulations reported in the following section rely on the simplest among the naive forecast rules, assuming firms forecast their competitors’ prices taking static expectations, $Q^F(t) = Q(t-1)$.

A second interesting by-product of non-stationarity is that firms neither

\textsuperscript{11} Indeed, non-stationarity even prevents the applicability of recursive least squares learning schemes.
can calculate their probability of bankruptcy from a stationary (i.e. ergodic) distribution\textsuperscript{12}, nor they can make Bayesian inference in updating it. Again, firms have to adopt some naive rule. A natural candidate is that firms assign to their probability of default an hazard function decreasing with time. In other words, as time goes by, conditioned to the simple fact that the firm survives the probability to incur in bankruptcy decreases. The figure 2 illustrates this case.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Individual ‘perceived’ probability of bankruptcy.}
\end{figure}

At time 0, a firm enters the market. The hazard function associated to its probability of bankruptcy is strictly less that one, and decreases as time goes by with a reverse logistic shape. After some time, the probability of bankruptcy reach asymptotically its inferior limit, which can be very close but strictly higher than zero. As we will discuss informally below, this hazard function could produce qualitatively interesting dynamics along the business cycle.

By now, it is necessary to stress that this assumption introduces an additional source of heterogeneity, as can be appreciated by noting that firms with different ages have also different probabilities to go bankrupt. Furthermore, due to the ‘entry-and-exit’ process the distribution of hazard rates over the population of firms changes potentially at any point in time. To keep track of the evolving hazard rate distribution within the economy is

\textsuperscript{12} For analytical convenience, the underlined price distribution almost universally considered in the literature when calculating the probability of default is the uniform distribution.
in general quite difficult. In particular, from a computationally point of view this amounts to add to the problem a set of additional state variables, each one capturing one of the relevant features (i.e. moments) of the firms’ distribution.

Since in this paper we are primarily concern with the long-run consequences of financial market imperfections, at this stage of the analysis we prefer to focus on a restricted version of the hazard function approach. Thus, in what follows we assume a common fixed probability - that is, all firms perceive their probability to incur in bankruptcy equals to the ‘asymptotic’ value of the hazard function - and we allow it to shift exogenously.

From the first order condition we obtain a standard Best Replay Function (BRF):

\[
P_j(t) = \frac{B \left(1 + r \frac{D(t, K)^{\xi - 1}}{1 - d\phi_j} \right)^{\frac{1}{\delta}}}{Q'(t)}
\]  

(9)

where \( B = \frac{\sigma \alpha}{\sigma - 1}, \) and \( \delta = 1 + \sigma(\alpha - 1). \)

As usual, each firm fixes its profit-maximizing price by choosing a point along its BRF. Optimal price is an increasing function both of the contractual interest rate on loans \( r, \) and of the marginal cost of bankruptcy \( d\phi_j: \) a higher probability of bankruptcy worsens the competitive position.

As a natural counterpart, we observe that the output associated with optimal price is given by:

\[
y_j(t) = \frac{B \left(1 + r \frac{D(t, K)^{\xi - 1}}{1 - d\phi_j} \right)^{\xi}}{e_j(t)^{\xi}}
\]  

(10)

where \( \epsilon = \frac{\sigma}{\delta} \) and \( \xi = 1 + \frac{\sigma}{\delta}(\alpha - 1). \) In line with standard results of the asymmetric information-based NKE (see e.g. Greenwald and Stiglitz, 1993), individual output is a decreasing function of \( r \) and of \( d\phi. \) In other words, a worsening of financial conditions has a negative effect on output.
4.3 The R&D lottery.

R&D investments, which generates innovative technologies, are modelled as uncertain activities, whose outcomes are governed by a two-stage stochastic process. At any time, each firm spend their retained profits to buy as many tickets of the lottery as possible. One can think of these tickets as wages to be paid to scientists – whose duty is that of discovering new productivity enhancement technologies\(^{13}\) – employed in the firm’s R&D department. Discovery and implementation of new technologies are uncertain activities themselves, in a sense that will become clearer below.

Let us now describe the two stages of the lottery in turn.

Stage 1. At time \(t+\tau\) each firm sets up an R&D department by hiring a certain number of scientists. How many scientists a firm can hire depends on the amount of financial resource the firm can count on. The scientists’ duty is that of transforming the money invested on them by the firm – that is, retained profits - in a productivity enhancing discovery.

Let \(I_j(t+\tau)\) be a Boolean random variable indicating firm \(j\)’s success in innovative R&D. It takes value 1 if a new technology has been found, 0 otherwise. The probability of \(I_j(t+\tau)\) to be 1 is given by:

\[
\mu_j(t+\tau) = \Pr\{I_j(t+\tau) = 1\} = 1 - \kappa \exp\left(\tau_j(t + \tau)\right)^\theta
\]

where \(\kappa\) and \(\theta\) are parameters, with \(\kappa > 1\) and \(0 < \theta < 1\). This latter condition means that search for new technologies is characterized by decreasing returns.

Stage 2. If \(I_j(t+\tau) = 1\), \(j\)’s productivity is given by:

\[
E\{e_j(t + \tau)\} = e_j(t - 1)(1 + \lambda)
\]

i.e. the firm experiences an increase of productivity on average equal to a percentage \(\lambda\), which is the mean value of a Poisson distributed stochastic process, with a variance equal to \(\nu^2\).

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\(^{13}\) Given the absence of physical capital, in our framework the expression ‘new technology’ should be better interpreted as a new form of organizing production.
5. Simulations.

The analytical setting presented above provides all the ingredients for an idealized characterization of the dynamic behaviour of an effectively and continuously changing economy with asymmetric information. In our world, individuals are endowed with a bounded ability to forecast the future, and interact strategically. Unfortunately, the complex structure of the composite stochastic process driving the system dynamics does not allow a 'closed form' analytical solution. Henceforth, the basic long-run properties of our economy should be inferred by resorting to simulations.

At time 0, firms are assumed to adopt the same technology and to face an identical market share, equal to 15 units of output. Initial individual (and aggregate, given the assumption of initial symmetry) price is fixed at 10. At the beginning of period 1 a vector of idiosyncratic ‘mean preserving spread’ technological shocks displaces firms from the homogeneous equilibrium. The stochastic model of section 3 is then employed to simulate the dynamic behaviour of the whole economy. At any stage of the market game, aggregate disposable income is given by real wages plus real profits from the previous period. The average increase in productivity $\lambda$ associated to successful R&D equals to an expected jump of 3.5% from the present technology, and it is common to all firms. Finally, as a working assumption we allow firms exited in time $t$ to be replaced by an equal number of firms with the ‘average’ technology in $t-1$.

Table 1 reports the starting value of relevant variables and parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial level of technology $e_j(0)$</td>
<td>4.53</td>
</tr>
<tr>
<td>Elasticity of substitution $\sigma$</td>
<td>3</td>
</tr>
<tr>
<td>‘Static’ returns $\alpha$</td>
<td>1.2</td>
</tr>
<tr>
<td>Fixed cost $F$</td>
<td>5</td>
</tr>
<tr>
<td>Increase in productivity $\lambda$</td>
<td>0.035</td>
</tr>
<tr>
<td>Number of firms $K$</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 1 – Values of relevant parameters and initial value of individual technology.

Simulations are performed with the goal of comparing two alternative situations with reference to 1) the ‘depth’ of financial markets, particularly
the market for loans; and 2) the ‘turbulence’ on the goods markets, here measured by alternative assumptions on the probability to incur in bankruptcy. There is a large body of evidence\textsuperscript{14} that lends support to the fact that Less-Developed Countries (LDCs) are characterized by a limited development of their financial industry, which translates into far from perfectly competitive conditions on the market for loans. Furthermore, it seems plausible to associate to underdeveloped economies a higher degree of ‘borrower’s risk’, that is of the risk of bankruptcy as it is perceived by managers. Our model permits precisely to analyze the mechanism through which these two effects are detrimental to real growth.

In figure 3 we plot the logarithm of the average growth path for two alternative parameterizations of the model\textsuperscript{15}. Averages have been obtained by taking the arithmetic mean from 100 simulations for each parameterized treatment. The dashed line represents the case of ‘cheap finance’, in which the interest rate on loans is equal to 2.25%, the marginal cost of bankruptcy is at 0.2, and the individual probability of bankruptcy is almost null ($\phi = 0.001$). The solid line illustrates the case of ‘expensive finance’, where $r$ is 5%, the probability of bankruptcy is at 2%, and the marginal cost of bankruptcy is 0.4. This latter case, if compared to the former one, is meant to capture the typical financial conditions of a LDC. In particular, the idea behind a higher real interest rate is that banking industry in LDCs is likely to be characterized by less competitive conditions in comparison to those in developed countries.

\textsuperscript{14} See King and Levine (1993).
\textsuperscript{15} Simulations have been run for a time span of 110 periods. The first 10 periods are not represented in order to get rid of transients.
Figure 3 – Average growth paths for two alternative parameterizations:
1) dashed line: \( r = 0.025; \phi = 0.001; d = 0.2 \)
2) solid line: \( r = 0.05; \phi = 0.02; d = 0.4. \)

Figure 3 highlights that a worsening of financial conditions decreases the average rate of growth. The reason for this result can be easily understood by resorting to equation (10). \textit{Ceteris paribus}, individual equilibrium output is negatively affected by an increase of the cost to finance production. According to our model, economies starting from an identical position but for their financial conditions will experience diverging growth paths.

Unfortunately, by averaging simulated paths we are not allowed to appreciate the interaction between business cycles and growth. To investigate this issue, consider figure 4 where we plot the logarithm of two selected simulated growth paths for the two alternative parameterizations.
Figure 4 – Selected simulated growth paths for the two alternative parameterizations:

1) dashed line: ‘cheap finance’
2) solid line: ‘expensive finance’.

The solid line, which represents the case of ‘expensive finance’, is characterized by a marked tendency for its trend to be permanently lowered after any downturn. In other words, whenever conditions on the financial markets worsen, economic downturns have long-run effects. Once again, the intuition for this result is straightforward: slight changes in revenues can have large effects on firms’ cash flow. In the presence of significant capital market imperfections, this leads to less expenditures in R&D which means a lower rate of productivity growth. As a result, the growth path of the economy is permanently lowered. In fact, according to our story it is precisely this effect that determines the divergence of the average growth paths between countries with developed financial markets and countries in which financial markets are underdeveloped.

As a last point, we spend some words on a question left open in our previous discussion, that is: how would our results be affected by a complete application of the hazard function hypothesis? Even in the absence of any analytical or simulated counterpart, some reasonable conjectures may be advanced. Other things equal, whenever a firm goes bankrupt the newcomer which replaces it operates with a relatively higher probability of default and consequently, due to equation (10), with a lower production
level. In other words, the decreasing hazard function hypothesis represents a means of persistence for downturns. Recall that the probability that a sufficient number of defaults trigger a downturn is higher for an economy with tight financial conditions. Since the ‘severity’ of a recession – here represented by the number of periods in which the output is below its trend permanently influences the long-run rate of growth, the longer it takes for the individual hazard functions to reach their lower asymptotic value, the more the long-run aggregate path is affected by any downturn.

6. Conclusions.

In this paper we have provided a model of growth with heterogeneous firms in which, due to asymmetric information in the financial markets, the Modigliani-Miller theorem does not hold. While in general neo-classical models of growth assume perfect capital markets, in our framework the way growth is financed matters. In particular, simulations allow us to appreciate the negative impact on the long-run rate of growth of an increase of the contractual interest rate on loans and of the probability of bankruptcy as perceived by risk-averse managers.
Appendix

In this appendix we provide a sketch of the proof for the statement that the aggregate price process is a non-stationary stochastic process. The proof proceeds in several steps.

First, remember that the aggregate price index is of the CES type:

$$Q_t = \left(\frac{1}{K} \sum_{j=1}^{K} P_j(t)^{1-\sigma}\right)^{-\frac{1}{\sigma}}$$

and that from equation (9) it follows that, for any given $Q(t)$, firm's $j$ price is a stochastic process $P_j(t) = \Gamma(D(t), e_j(t))$.

Second, making use of the fact that the probability to win the R&D lottery is strictly less than one, one can easily see that the number of firms innovating each period is a random variable $\varphi_t$, with $\varphi \sim (1, K)$. It follows that the aggregate price index can be written as:

$$Q(t) = \left(\frac{1}{K} \sum_{j=1}^{K} \varphi_t (t-1)^{1-\sigma} + \frac{1}{K} \sum_{j=\varphi+1}^{K} \Gamma_j(D(t), e_j(t))\right)^{-\frac{1}{\sigma}}$$

where, as we said, $e_j(t)$ is a stochastic variable as well.

Finally, from the arguments above it follows that $Q(t)$ can be expressed the non-stationary sum of a composite stochastic process:

$$Q(t) = Q(0) + \sum_{\tau=0}^{t} u(\varphi(\tau), \Gamma(\tau)).$$
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